Homework Assignment 5 in Differential Equations, MATH308-FALL 2013

due September 18, 2013

Topics covered : *Wronskian, fundamental set of solutions of linear homogeneous equations of second order, linear homogeneous equations of second order with constant coefficients: the case of two distinct real roots of the characteristic polynomial (corresponds to sections 3.2, and 3.1 in the textbook).*

1. (a) Find the Wronskian of the functions $y_1(t) = t^{-1}\cos(2\ln t)$, $y_2(t) = t^{-1}\sin(2\ln t)$, $t > 0$;

   (b) Show that functions $y_1(t)$ and $y_2(t)$ from the previous item are solutions of the differential equation

   $$t^2y'' + 3ty' + 5y = 0, \quad t > 0 \tag{1}$$

   (c) Show that the functions $y_1(t)$ and $y_2(t)$ from item a) constitute the fundamental set of solutions of the equation (1) and find the general solution of the equation (1)

   (d) Find the solution of (1) satisfying the initial conditions $y(1) = 2$, $y'(1) = -4$.

2. Given the differential equation

   $$y'' + 3y' - 10y = 0. \tag{2}$$

   (a) Find the general solution of this equation;

   (b) Find the solution of the same equation satisfying the initial condition

   $$y(0) = \alpha, y'(0) = 1; \tag{3}$$

   (c) Find all values of α so that the solution of the initial value problem (2)-(3) approach zero as $t \to +\infty$;

   (d) Find all values of α so that the solution of the initial value problem (2)-(3) approach zero as $t \to -\infty$.

3. (**bonus-20 points**) Read the formulation and the proof of Theorem 3.2.6 on the page 153 (Abel's Theorem). Then, using this theorem, find the value of $W(y_1, y_2)(5)$, if the functions $y_1(t)$ and $y_2(t)$ constitute a fundamental set of solutions of the equation $ty'' + 3y' + \sin(e^t)y = 0$, $t > 0$, such that $W(y_1, y_2)(3) = 4$ (for this you do not actually need to find $y_1(t)$ and $y_2(t)$).