

Homework Assignment 9 in Differential Equations, MATH308

due April 18, 2012

Sections covered 7.5(partially)-7.6

1. Solve the following initial value problem

$$\begin{aligned}x_1' &= x_2 + x_3 \\x_2' &= x_1 + x_3 \\x_3' &= x_1 + x_2\end{aligned}$$

subject to $x_1(0) = -1$, $x_2(0) = 4$, $x_3(0) = 0$.

2. Find the fundamental matrix and the general solution for the following system of equations:

$$(a) \begin{aligned}x_1' &= 6x_1 - x_2 \\x_2' &= 5x_1 + 2x_2\end{aligned}$$

$$(b) \mathbf{x}' = \begin{pmatrix} 1 & -1 & 2 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix} \mathbf{x}$$

3. Solve the following initial value problem

$$(a) \mathbf{x}' = \begin{pmatrix} -3 & -1 \\ 2 & -1 \end{pmatrix} \mathbf{x}, \quad \mathbf{x}(0) = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

$$(b) \mathbf{x}' = \begin{pmatrix} 1 & -12 & -14 \\ 1 & 2 & -3 \\ 1 & 1 & 2 \end{pmatrix} \mathbf{x}, \quad \mathbf{x}(0) = \begin{pmatrix} 4 \\ 6 \\ -7 \end{pmatrix}$$

4. (**bonus -20pts**) Given the system of differential equations $\mathbf{x}' = A\mathbf{x}$ where

$$A = \begin{pmatrix} \alpha & -1 \\ -1 & \alpha \end{pmatrix}.$$

Here α is a real parameter.

- Determine the eigenvalues of A .
- Find the critical values of α where the qualitative nature of the phase portrait of the system changes. What type of equilibrium point is the origin for each α ?
- Sketch the phase portrait for $\alpha = 0$, $\alpha = 1$ and $\alpha = -1$.