1- Homework assignment #9, Solutions, MATH 308-505, ...

1(a) $3y''-6y'+6y=e^x\sec x \iff y''-2y'+2y=\underbrace{\frac{1}{3}e^x\sec x}_{g(x)}$

Solution: • Homogeneous equation:

$$3y''-6y'+6y=0 \iff$$

$$y''-2y'+2y=0$$

• The characteristic equation is

$$r^2-2r+2=0$$

$$D/4=1-2=-1 \Rightarrow$$

$$r_{1,2}=1\pm i$$

• The gen. solution of homogeneous equation is

$$y(x)=C_1e^x\cos x+C_2e^x\sin x$$

$$\Downarrow$$

We look for a solution of nonhom. eq. in the form

$$y(x)=u_1(x)e^x\cos x+u_2(x)e^x\sin x$$

such that

$$\begin{cases} u_1'(x)e^x\cos x+u_2'(x)e^x\sin x=0 & \text{(Eq.1)}\\ u_1'(x)(e^x\cos x-e^x\sin x)+u_2'(x)(e^x\sin x+e^x\cos x)=\frac{1}{3}e^x\sec x & \text{(Eq.2)}\\ -u_1'(x)e^x\sin x+u_2'(x)e^x\cos x=\frac{1}{3}e^x\sec x & \text{(Eq.2')}\end{cases}$$

$-(\text{Eq 1})+(\text{Eq2}) \Rightarrow$

• $\cos x(\text{Eq 1})-\sin x(\text{Eq 2'}) \Rightarrow u_1'(x)\cancel{e^x}=-\frac{1}{3}\cancel{e^x}\tan x \Rightarrow u_1'(x)=-\frac{1}{3}\tan x$

$$\Rightarrow u_1(x)=-\frac{1}{3}\int\frac{\sin x}{\cos x}dx+C_1=\frac{1}{3}\int\frac{du}{u}+C_1=\frac{1}{3}\ln|u|+C_1=\frac{1}{3}\ln|\cos x|+C_1$$

$u=\cos x$

$du=-\sin x\,dx$

• $\sin x(\text{Eq 1})+\cos x(\text{Eq 2'}) \Rightarrow u_2'(x)\cancel{e^x}=\frac{1}{3}\cancel{e^x} \Rightarrow u_2'(x)=\frac{1}{3} \Rightarrow$

$$u_2(x)=\frac{1}{3}x+C_2 \Rightarrow$$

$$\boxed{y(x)=\left(\frac{1}{3}\ln|\cos x|+C_1\right)e^x\cos x+\left(\frac{1}{3}x+C_2\right)e^x\sin x=\frac{1}{3}e^x\ln|\cos x|\cos x+\frac{1}{3}xe^x\sin x+C_1e^x\cos x+C_2e^x\sin x}$$