Chapter 1-Polynomials and Modeling

Question 1.1

Algebraic Method: Slope of a line is the amount of change in y when x increases by one unit. For this function,

$$m = \frac{\Delta f(x)}{\Delta x} = \frac{-2}{3}$$

If x decreases by $9 \Rightarrow \quad \Delta x = -9$, so

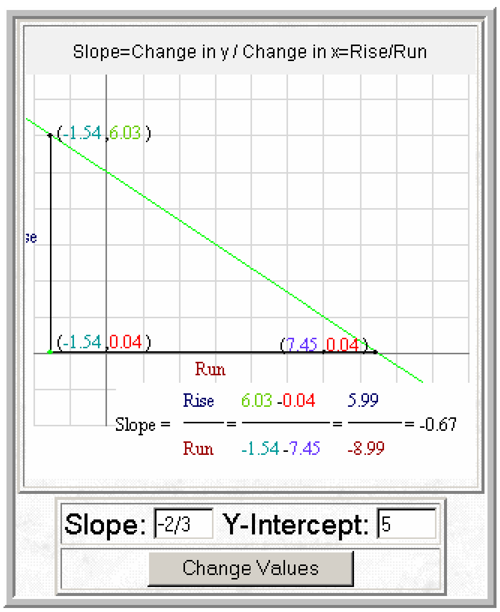
$$\frac{\Delta f(x)}{-9} = \frac{-2}{3}$$

Cross-multiplying gives

$$3(\Delta f(x)) = 18$$
$$\Delta f(x) = {}^{18}\!/_{3} = 6$$

Therefore, if x decreases by 9, $f(x)$ increases by 6.

Applet Method: Using the Lines and Slope Applet, you can enter the given function, using its slope and *y*-intercept. Because we want the "run" to change by -9, drag the points so that this happens. Then notice what happens to the "rise" (or change in function value), as shown below for a "run" of about -9.



Question 1.2

Algebraic Method: In order to find profit, *P(x)*, you need the cost and revenue functions, *C(x)* and *R(x)*. Total costs are given by $C(x) = mx + b$, so we can organize the given cost information as ordered pairs of the form, $(x, C(x))$. Thus, we have (50, 550) and (100, 650) as two points on our linear cost function. Using these points to find the slope gives

$$m = \frac{\Delta C(x)}{\Delta x} = \frac{650 - 550}{100 - 50} = \frac{100}{50} = 2$$

So, $C(x) = 2x + b$. Plugging in one point, say (50, 550), allows us to solve for *b*.

$$\begin{aligned} C(50) = 2(50) + b &= 550 \\ 100 + b &= 550 \\ b &= 450 \end{aligned}$$

Therefore, $C(x) = 2x + 450$.

Because the fish bowls sell for \$5.00 each, we find the revenue function to be $R(x) = 5x$. Thus, the profit function is given by

$$\begin{aligned} P(x) &= R(x) - C(x) \\ &= 5x - (2x + 450) \\ &= 3x - 450 \end{aligned}$$

Question 1.3

Algebraic Method

a. A linear demand function has the form $D(x) = p = mx + b$, so we can organize the given information as ordered pairs of the form $(x, p) =$ (quantity, price). Thus, we have (500, 30) and (100, 40) as two points on our linear demand function. Finding the slope between the two points gives

$$m = \frac{\Delta p}{\Delta x} = \frac{40 - 30}{100 - 500} = \frac{10}{-400} = -\frac{1}{40}$$

Using one of the points and the point-slope form of a line shows demand to be

$$\begin{aligned} p - 30 &= {}^{-1}\!/_{40}(x - 500) \\ p - 30 &= ({}^{-1}\!/_{40})x + {}^{50}\!/_{4} \\ p &= {}^{-1}\!/_{40}\,x + {}^{170}\!/_{4} \end{aligned}$$

b. The highest price consumers are willing to pay is indicated by the *y*-intercept of the demand function. Therefore, the highest price consumers will pay is \$170/4 = \$42.50.

Question 1.4

Algebraic Method: Linear depreciation is given by $V(t) = mt + b$. Thus, we can organize the given information as ordered pairs of the form $(t, V(t)) =$ (time, value). Allowing the year 1999 to be represented by $t = 0$, we have (0, 20500) and (3, 13285) as two points on our linear depreciation function. Finding the slope between the two points gives

$$m = \frac{\Delta V(t)}{\Delta t} = \frac{13285 - 20500}{3 - 0} = \frac{-7215}{3} = -2405$$

Because *b* represents the original value of the car ($V(0) = m(0) + b = b$), we have the value of the car given as $V(t) = -2405t + 20500$.

In 2006, t = 7, so we have $V(7) = -2405(7) + 20500 = 3665$. Therefore, the car will be worth $3665 in the year 2006.

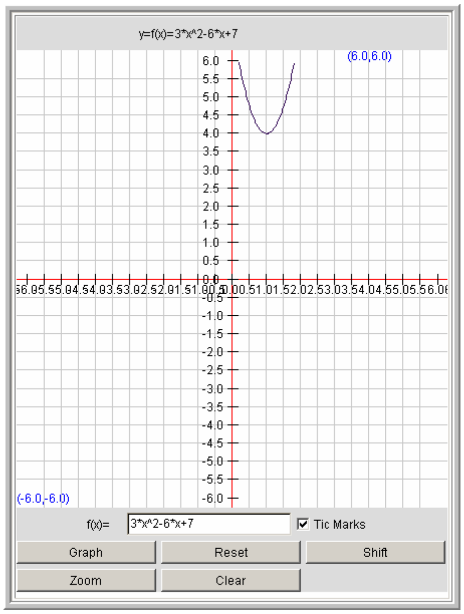
Question 1.5

Algebraic Method: The vertex of a parabola ($f(x) = ax^2 + bx + c$) is given by $(-b/2a, f(-b/2a))$. For, $f(x) = 3x^2 - 6x + 7$, a = 3, b = -6, and c = 7. Thus, the x-coordinate of the vertex is

$$x = \frac{-b}{2a} = \frac{-(-6)}{2(3)} = 1$$

and the y-coordinate will be $y = f(1) = 3(1)^2 - 6(1) + 7 = 4$.
Therefore, the vertex of the parabola is at (1, 4).

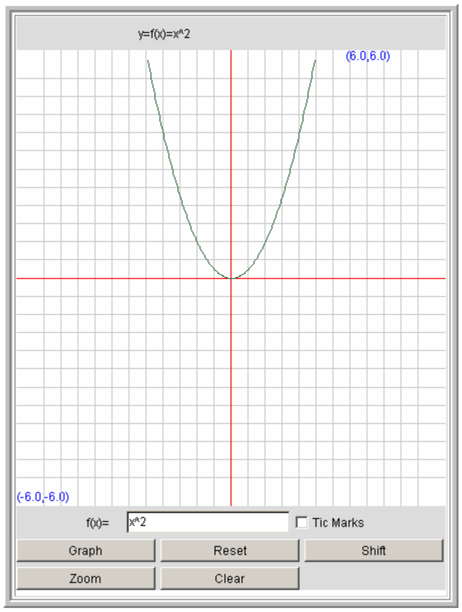
Applet Method: You can plot the quadratic function using the Plotting Applet and find the vertex on the graph, as shown.



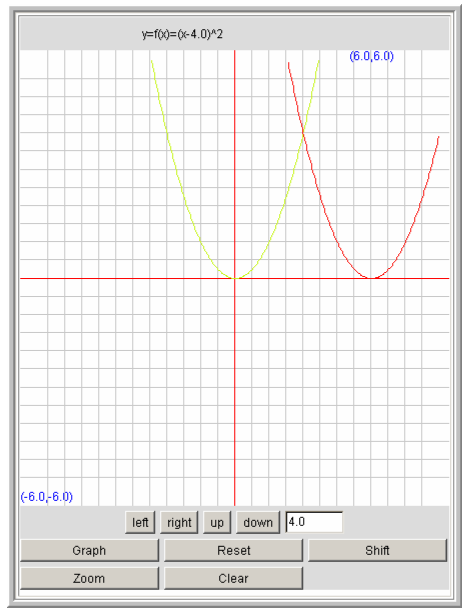
Question 1.6

Applet Method: You can use the Plotting Applet to see each of the changes made to the original graph.

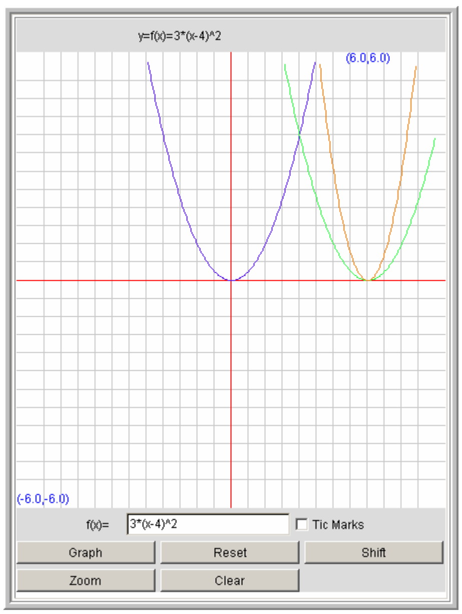
Start with the graph of $f(x) = x^2$.



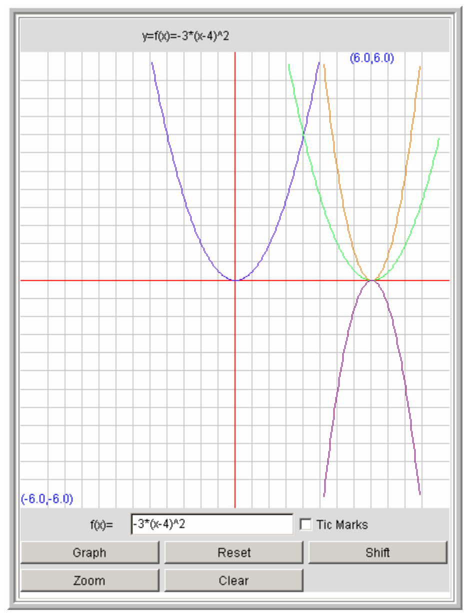
$f_2(x) = (x-4)^2$ shifts $f(x)$ to the right 4 units.



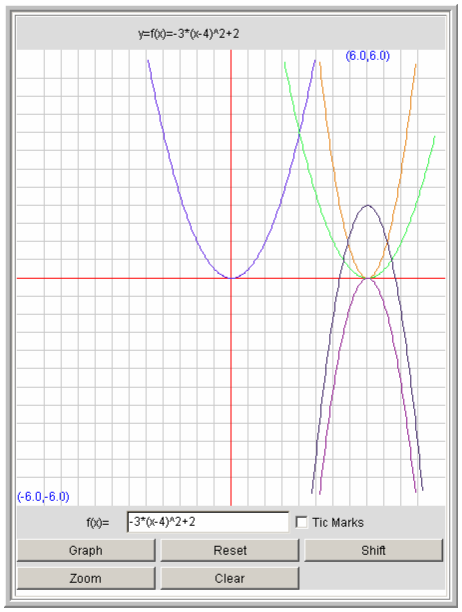
$f_3(x) = 3(x-4)^2$ vertically expands $f_2(x)$ by a factor of 3.



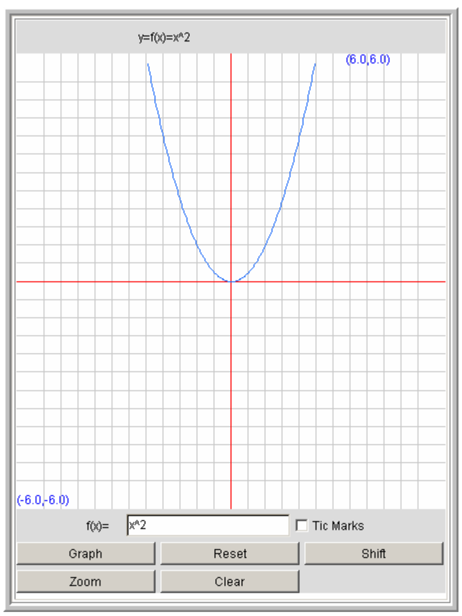
$f_4(x) = -3(x-4)^2$ vertically reflects $f_3(x)$ over the x-axis.

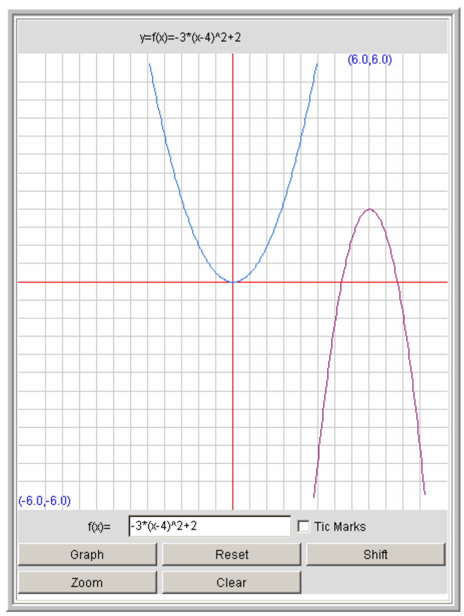


$f_5(x) = -3(x-4)^2 + 2$ shifts $f_4(x)$ up 2 units.



Thus, $g(x)$ will be different from $f(x)$ because it will be shifted 4 units to the right, vertically expanded by a factor of 3, reflected over the *x*-axis, and then shifted up 2 units.





Question 1.7

Algebraic Method: Profit is given by $P(x) = R(x) - C(x)$. In order to find revenue, $R(x)$, we need the selling price of the item. Because no set selling price is given, we use the demand function which gives the price consumers are willing to pay for *x* items. Therefore,

$$R(x) = px = (-2x + 50)x = -2x^2 + 50x$$

Now,

$$
\begin{aligned}
P(x) &= R(x) - C(x) \\
&= -2x^2 + 50x - (30x + 40) \\
&= -2x^2 + 50x - 30x - 40 \\
&= -2x^2 + 20x - 40
\end{aligned}
$$

Because the coefficient of x^2 is –2 < 0, the graph of the profit function will be a parabola opening downward, meaning the maximum profit occurs at the point of the vertex.

The number of items the company must make and sell in order to maximize profits is given by the *x*-coordinate of the vertex. Thus, the company should make and sell

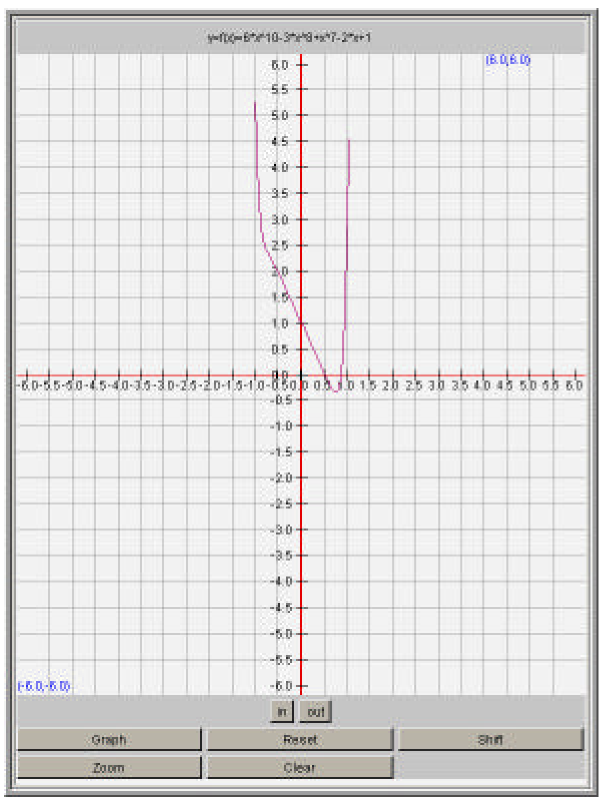
$$x = \frac{-b}{2a} = \frac{-20}{2(-2)} = 5$$

items to maximize it's profits.

Question 1.8

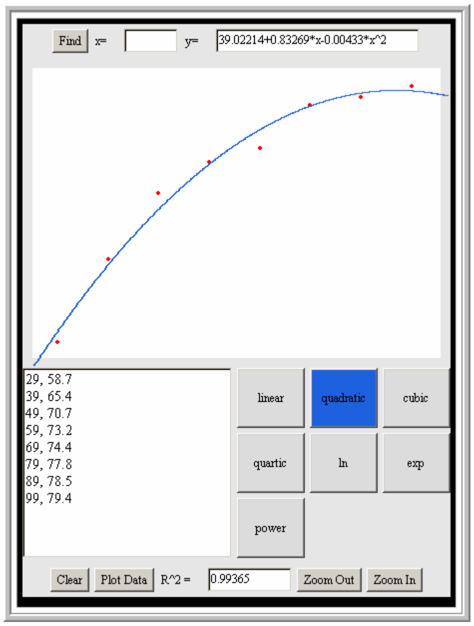
<u>Algebraic Method:</u> $f(x) = 6x^{10} - 3x^8 + x^7 - 2x + 1$ is an even degree polynomial since the highest power of *x* is equal to 10. Moreover, since $a_n = 6 > 0$ we know as $x \to \infty, f(x) \to \infty$ and as $x \to -\infty, f(x) \to \infty$.

<u>Applet Method:</u> You can plot the function in the Plotting Applet and notice that both ends of the function grow larger as the *x* values become both more negative and more positive.



Question 1.9

Applet Method: We will let the independent variable, *x*, represent the number of years since 1900 (i.e., *x* = 29 represents the year 1929) and the dependent variable, *y*, represent the life expectancy of females respectively. Inputting the data into the Modeling Applet and clicking on the quadratic regression button we get

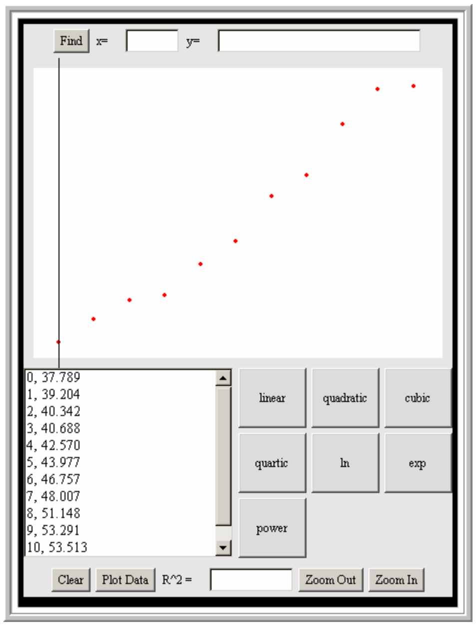


As you can see at the top of the applet, the equation of the best-fitting quadratic function to this data is

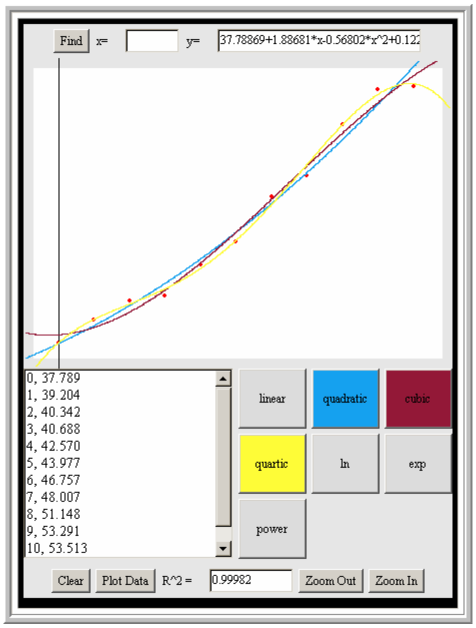
$$y = -0.00433x^2 + 0.83269x + 39.02214$$

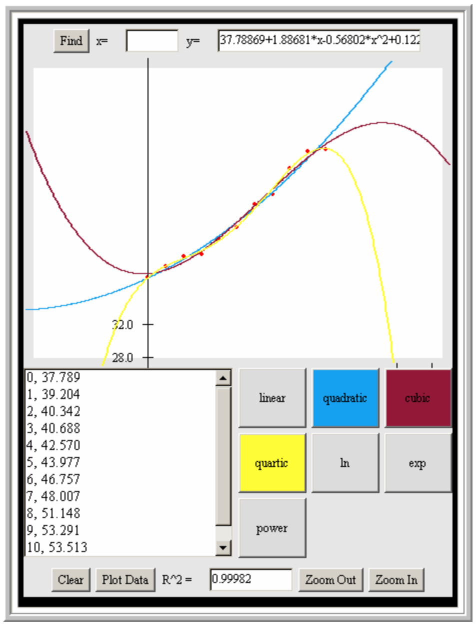
Question 1.10

Applet Method: We will let the independent variable, *x*, represent the number of years since 1990 (i.e., *x* = 0 represents the year 1990) and the dependent variable, *y*, represent the median income (in thousands of dollars) (i.e. *y* = 37.789 represents $37, 789), respectively. We will first input the data into the Modeling Applet and look at a scatterplot of the data.



Looking at the data, it seems to have some curves to it, suggesting that a linear model is probably not the best-fitting polynomial to the data. However it is hard to rule out any of the other three polynomial models. Thus, we will take a look at the quadratic, cubic, and quartic models and consider how they fit the data given and how they will fit data outside of the given domain.





All of the models increase in the domain given and fit the data points relatively well, as shown in the left figure above. However, one would have expected (1) the incomes to be lower before 1990 (not higher, as the cubic model suggests in the above right figure) and (2) the incomes to increase somewhat after 2000 (not immediately decrease, as the quartic model suggests in the above right picture). Thus, it seems that the best-fitting polynomial model to this data would be the quadratic model.

Chapter 2 – Exponentials and Logarithms

Question 2.1

The successive ratios are given below:

$$\frac{f(4)}{f(3)} = \frac{51}{42} \approx 1.1243 \quad \frac{f(5)}{f(4)} = \frac{62}{51} \approx 1.2157 \quad \frac{f(6)}{f(5)} = \frac{75}{62} \approx 1.2097$$

$$\frac{f(7)}{f(6)} = \frac{90}{75} = 1.2 \quad \frac{f(8)}{f(7)} = \frac{109}{90} \approx 1.2111 \quad \frac{f(9)}{f(8)} = \frac{133}{109} \approx 1.2202$$

Since all of the ratios are approximately 1.2 > 0, we can conclude that the data represent an exponential growth function with a base of 1.2.

Question 2.2

<u>Algebraic Method:</u>

$$4^x - 13\cdot 2^x = -36$$

$$(2^2)^x - 13\cdot 2^x = -36 \qquad \text{(Get the bases the same.)}$$

$$2^{2x} - 13\cdot 2^x = -36$$

$$2^{2x} - 13\cdot 2^x + 36 = 0$$

$$(2^x - 4)(2^x - 9) = 0$$

$$2^x - 4 = 0 \quad \text{or} \quad 2^x - 9 = 0$$

$$2^x = 4 \qquad\qquad 2^x = 9$$

$$2^x = 2^2 \qquad\qquad \ln 2^x = \ln 9$$

$$x = 2 \qquad\qquad x\ln 2 = \ln 9$$

$$x = \frac{\ln 9}{\ln 2} \approx 3.1699$$

Question 2.3

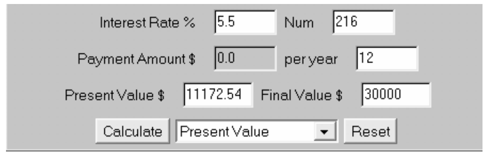
<u>Algebraic Method</u>: $A = P(1+\frac{r}{n})^{nt}$ is the equation used for finding the amount of money in an account where the interest is compounded during a specified time period. We are given that $r = 0.055$, $n = 12$ (monthly), $A = \$30{,}000$, and $t = 18$. We are looking for the value of P.

$$30000 = P\left(1+\frac{0.055}{12}\right)^{12(18)} = P\left(\frac{2411}{2400}\right)^{216}$$

$$P = \frac{30000}{\left(\frac{2411}{2400}\right)^{216}} \approx 11172.54269$$

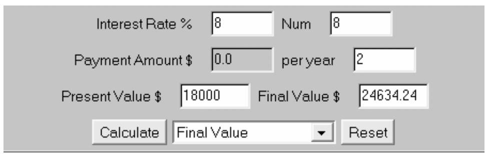
Thus, Dave should deposit \$11,172.55 if he wants to ensure that he will have \$30,000 at the end of the 18 years.

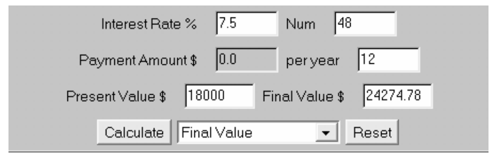
Applet Method: Fill in the given entries and calculate the Present Value, as shown below.



Question 2.4

Applet Method: For both banks the Present Value is 18,000 and the time is 4 years. For Bank Two, the interest rate is 8%, per year is 2 (semi-annually) and so Num = 2×4 = 8. For Bank One, the interest rate is 7.5%, per year is 12 (monthly) and therefore Num = 12×4=48. The final values are calculated with the compound interest applet shown below:





Bank Two has $24,634.24.

Bank One has $24,274.78

So Bank Two has 24,634.24 – 24,274.78 = 359.47 more than Bank One.

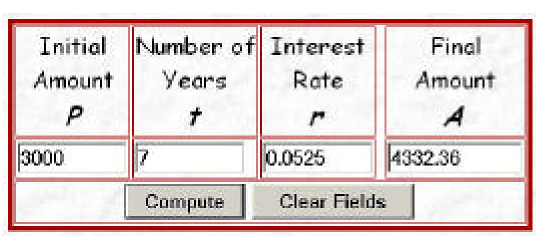
Question 2.5

Algebraic Method: For accounts compounded continuously, $A = Pe^{rt}$. Here, P = $3000, r = 0.0525, and t = 7. Looking for A we get

$$A = Pe^{rt} = 3000e^{(0.0525)(7)} \approx 4332.359396$$

So, you will have approximately $4332.36 in the account after 7 years.

Applet Method: Enter the values into the Continuous Compounding Applet and click Computer.



Question 2.6

Algebraic Method:

$$\ln(\ln 2x) = 0 \Rightarrow e^0 = \ln 2x$$
$$\Rightarrow 1 = \ln 2x$$

$$\ln 2x = 1 \Rightarrow e^1 = 2x$$
$$\Rightarrow x = \frac{e}{2} \approx 1.359$$

Remember to check that the *x* value is in the domain of the problem ($2x > 0 \Rightarrow x > 0$).

Question 2.7

Algebraic Method: We are given that *S* = 215 and we need to solve for *p*:

$$215 = 150e^{0.004p} \quad \rightarrow \quad \frac{43}{30} = e^{0.004p}$$
$$\ln(\tfrac{43}{30}) = \ln(e^{0.004p}) = 0.004p(\ln e) = 0.004p(1)$$
$$p = \frac{\ln(\tfrac{43}{30})}{0.004} \approx 90.00068351$$

Therefore, the store should sell the DVD players for approximately $90.

Question 2.8

Algebraic Method: Using logarithm rules,

$$\log_b\left(\frac{21}{25}\right) = \log_b 21 - \log_b 25 = \log_b(3\cdot 7) - \log_b\left(5^2\right) = \log_b 3 + \log_b 7 - 2(\log_b 5)$$

Thus, with the given information,

$$\log_b\left(\frac{21}{25}\right) = 1.5850 + 2.8074 - 2(2.3219) = -.2516$$

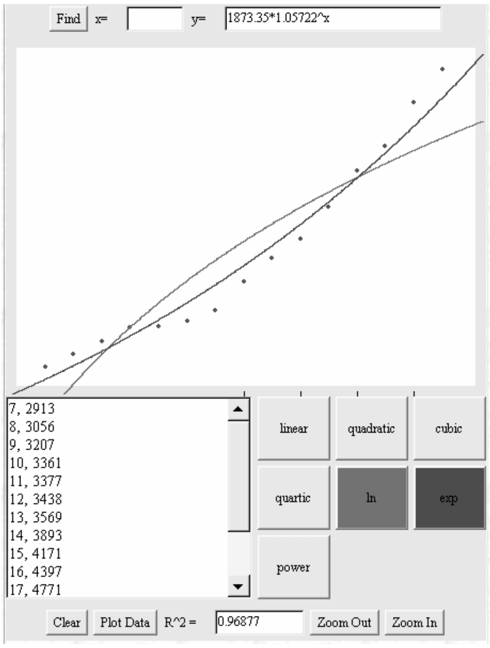
Question 2.9

Applet Method: Letting *x* = 7 represent the year 1987, our data becomes

x	7	8	9	10	11	12	13	14
y	2913	3056	3207	3361	3377	3428	3569	3893

x	15	16	17	18	19	20	21
y	4171	4397	4771	5181	5469	5983	6360

Note: You would not want to let *x* = 0 represent the year 1987, as the point (0, 2913) would not be able to be part of a logarithmic function because of the domain of the function. You can plot the data in the Modeling Applet and find both the best-fitting exponential and logarithmic models and their R^2 values, as shown below.



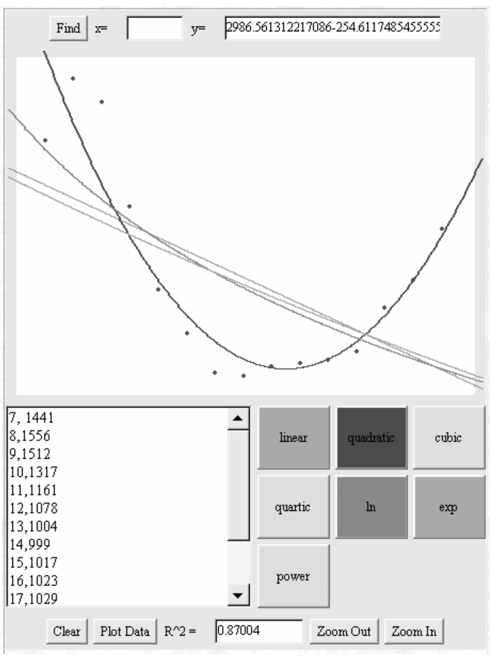
Exp fit has $y = 1873.35 \times (1.05722)^x$ with R^2 = .96877

Ln fit has $y = -3448.21 + 2960.48 \ln x$ with R^2 = .84483

By looking at both models with the data, it appears that the exponential model is a better fit to the data

Question 2.10

<u>Applet Method</u> Let x be the number of years after 1980. Enter the data into the applet and choose linear, quadratic, ln and exp fits. Only the quadratic model comes close to the shape of the data.



Chapter 4 – Rates of Change

Question 4.1

Algebraic Method: The average rate of unemployment is given by

$$\frac{\text{Change in Unemployment}}{\text{Change in Time}} = \frac{37097 - 20907}{11 - 4} = \frac{16190}{7} \approx 2312.857$$

Thus, the unemployment rate was increasing by approximately 2313 people per month between April and November.

Question 4.2

Algebraic Method: The average rate of change is given by

$$\frac{\Delta y}{\Delta x} = \frac{2.1 - 2.37}{68 - 16} = \frac{-0.27}{52} \approx -0.00519$$

Therefore, between 9/17/02 and 11/8/02, the rate for 1 year CDs was decreasing at 0.52% per day.

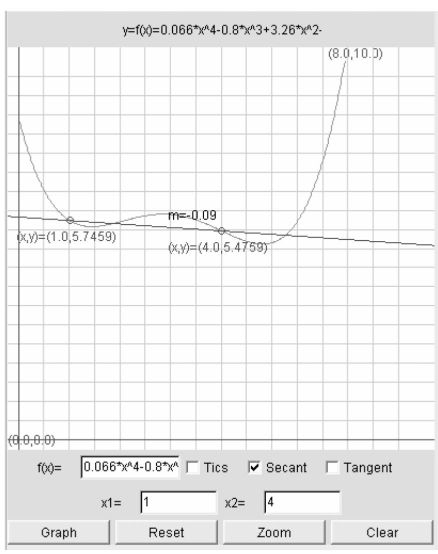
Question 4.3

Algebraic Method: 1997 will correspond to $x = 1$ and $f(1) = 5.746$. The year 2000 will correspond to $x = 4$ and $f(4) = 5.476$. The average rate of change is then

$$\frac{\Delta y}{\Delta x} = \frac{f(4) - f(1)}{4 - 1} = \frac{5.476 - 5.746}{3} \approx -0.09$$

Since *x* is in thousands of layoff events, the rate of change is -90 layoffs per year.

Applet Method The slope of the secant line is the average rate of change. Using the Secant/Tangent Applet we have



Question 4.4

Algebraic Method: The instantaneous rate of change of $f(x) = 3/x$ at x = 2 is

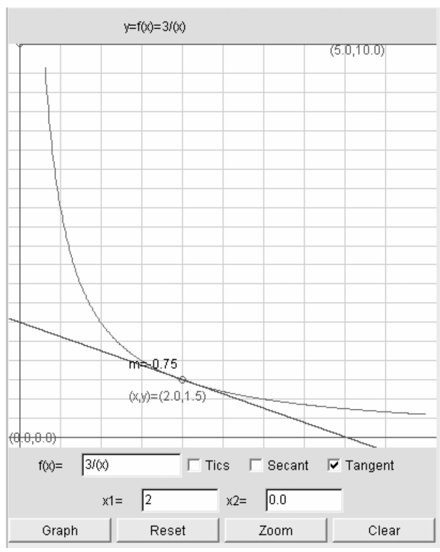
$$\lim_{h\to 0}\frac{f(2+h)-f(2)}{h} = \lim_{h\to 0}\frac{\left(\frac{3}{2+h}\right)-\left(\frac{3}{2}\right)}{h} = \lim_{h\to 0}\frac{\left[\frac{3(2)-3(2+h)}{2(2+h)}\right]}{h}$$

$$= \lim_{h\to 0}\left[\frac{6-6-3h}{2(2+h)}\right]\left[\frac{1}{h}\right] = \lim_{h\to 0}\left[\frac{-3h}{2(2+h)}\right]\left[\frac{1}{h}\right]$$

$$= \lim_{h\to 0}\frac{-3}{2(2+h)} = \frac{-3}{2(2+0)}$$

$$= \frac{-3}{4}$$

Applet Method We can find the instantaneous rate of change from the slope of the tangent line on the Secant/Tangent Applet:



Question 4.5

Algebraic Method: The slope of the tangent line to $f(x) = \sqrt{x-2}$ at x = 3 is equal to the instantaneous rate of change of $f(x)$ at x = 3:

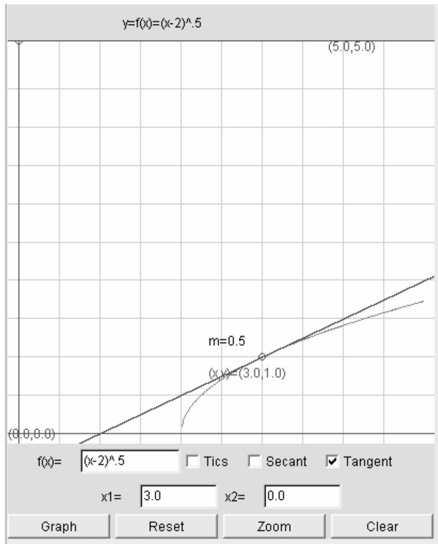
$$m_{\tan} = \lim_{h\to 0}\frac{f(3+h)-f(3)}{h} = \lim_{h\to 0}\frac{\sqrt{(3+h)-2}-\sqrt{3-2}}{h}$$

$$= \lim_{h\to 0}\frac{\sqrt{(3+h)-2}-1}{h}\frac{\left(\sqrt{(3+h)-2}+1\right)}{\left(\sqrt{(3+h)-2}+1\right)} = \lim_{h\to 0}\frac{\left(\sqrt{(3+h)-2}\right)^2+\left(\sqrt{(3+h)-2}\right)-\left(\sqrt{(3+h)-2}\right)-1}{h\left(\sqrt{(3+h)-2}+1\right)}$$

$$= \lim_{h\to 0}\frac{(3+h-2)-1}{h\left(\sqrt{(3+h)-2}+1\right)} = \lim_{h\to 0}\frac{h}{h\left(\sqrt{(3+h)-2}+1\right)} = \lim_{h\to 0}\frac{1}{\left(\sqrt{(3+h)-2}+1\right)}$$

$$= \frac{1}{\sqrt{3+0-2}+1} = \frac{1}{2}$$

Applet Method We can find the slope of the tangent line using the Secant/Tangent Applet:



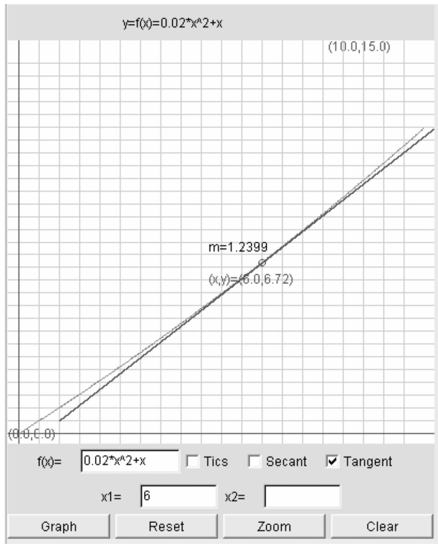
Question 4.6

Algebraic Method: The instantaneous rate of $R(x) = 0.02x^2 + x$ at $x = 6$ is

$$\begin{aligned}
\lim_{h\to 0}\frac{R(6+h)-R(6)}{h} &= \lim_{h\to 0}\frac{\left[0.02(6+h)^2+(6+h)\right]-\left[0.02(6)^2+6\right]}{h}\\
&= \lim_{h\to 0}\frac{\left[0.02\left(36+12h+h^2\right)+6+h\right]-\left[0.02(36)+6\right]}{h}\\
&= \lim_{h\to 0}\frac{0.02(36)+0.02(12h)+0.02h^2+6+h-0.02(36)-6}{h}\\
&= \lim_{h\to 0}\frac{0.02h^2+1.24h}{h} = \lim_{h\to 0}\frac{h(0.02h+1.24)}{h} = \lim_{h\to 0}\quad 0.02h+1.24\\
&= 0.02(0)+1.24 = 1.24
\end{aligned}$$

This means that when 6 motorcycles are sold, revenue is increasing by $1,240 per motorcycle.

Applet Method Using the Secant/Tangent Applet, the slope of the tangent line at $x = 6$ is 1.24:



Question 4.7

Algebraic Method: In order to find the equation of a line, you need a point on the line and the slope of the line. The tangent line to $f(x) = 3x^2 - 1$ at $x = -2$ will touch $f(x)$ at the point $(-2, f(-2)) = (-2, 11)$, giving us a point on the line we are looking to find. The slope of the tangent line is given by

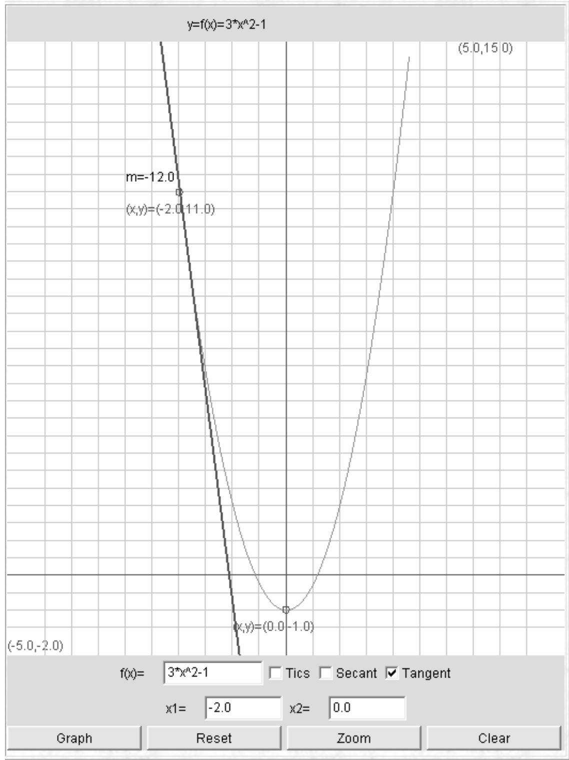
$$m_{\tan} = \lim_{h\to 0} \frac{f(-2+h) - f(-2)}{h} = \lim_{h\to 0} \frac{\left[3(-2+h)^2 - 1\right] - 11}{h}$$

$$= \lim_{h\to 0} \frac{\left[3\left(4 - 4h + h^2\right) - 1\right] - 11}{h} = \lim_{h\to 0} \frac{12 - 12h + 3h^2 - 1 - 11}{h}$$

$$= \lim_{h\to 0} \frac{-12h + 3h^2}{h} = \lim_{h\to 0} \frac{h(-12 + 3h)}{h} = \lim_{h\to 0} \quad -12 + 3h$$

$$= -12 + 3(0) = -12$$

Applet Method The slope of the tangent line can also be found using the Secant/Tangent Applet,



So, with m = –12 and the point (-2, 11), we can find the equation of the tangent line using the point-slope formula:

$$\begin{aligned} y - 11 &= -12(x - (-2)) \\ y - 11 &= -12x - 24 \\ y &= -12x - 13 \end{aligned}$$

Question 4.8

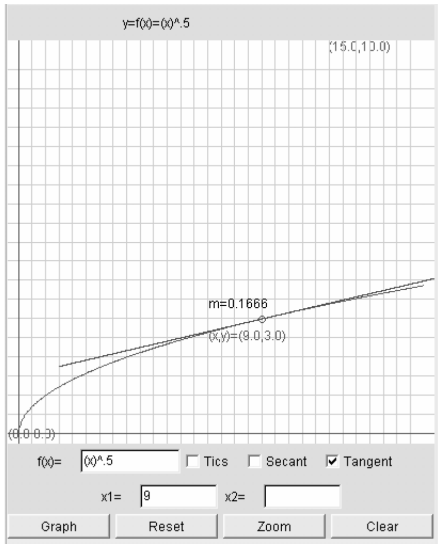
Algebraic Method: In order to find the equation of a line, you need a point on the line and the slope of the line. The tangent line to $g(x)$ at x = 9 will touch $g(x)$ at the point $(9, g(9)) = (9,3)$, giving us a point on the line we are looking to find. The slope of the tangent line is given by

$$m_{\tan} = \lim_{h\to 0} \frac{g(9+h)-g(9)}{h} = \lim_{h\to 0} \frac{\sqrt{9+h}-3}{h} \frac{\left(\sqrt{9+h}+3\right)}{\left(\sqrt{9+h}+3\right)} = \lim_{h\to 0} \frac{\left(\sqrt{9+h}\right)^2 + 3\sqrt{9+h} - 3\sqrt{9+h} - 9}{h\left(\sqrt{9+h}+3\right)}$$

$$= \lim_{h\to 0} \frac{(9+h)-9}{h\left(\sqrt{9+h}+3\right)} = \lim_{h\to 0} \frac{h}{h\left(\sqrt{9+h}+3\right)} = \lim_{h\to 0} \frac{1}{\left(\sqrt{9+h}+3\right)}$$

$$= \frac{1}{\left(\sqrt{9+0}+3\right)} = \frac{1}{6}$$

Applet Method The Secant/Tangent Applet can find the approximate slope,



So, with m = 1/6 and the point (9, 3), we can find the equation of the tangent line using the point-slope formula:

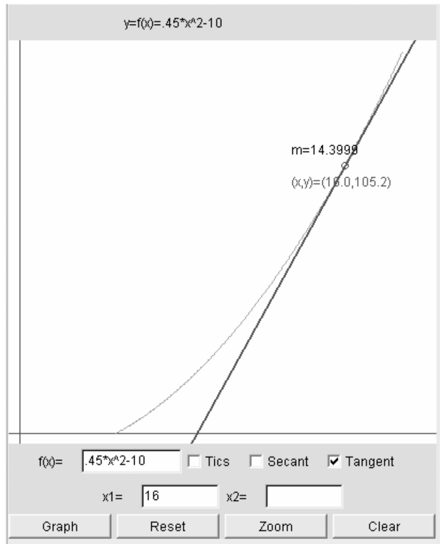
$$y - 3 = \tfrac{1}{6}(x - 9)$$
$$y - 3 = \tfrac{1}{6}x - \tfrac{9}{6}$$
$$y = \tfrac{1}{6}x + \tfrac{3}{2}$$

Question 4.9

Algebraic Method: The slope of the tangent line will be

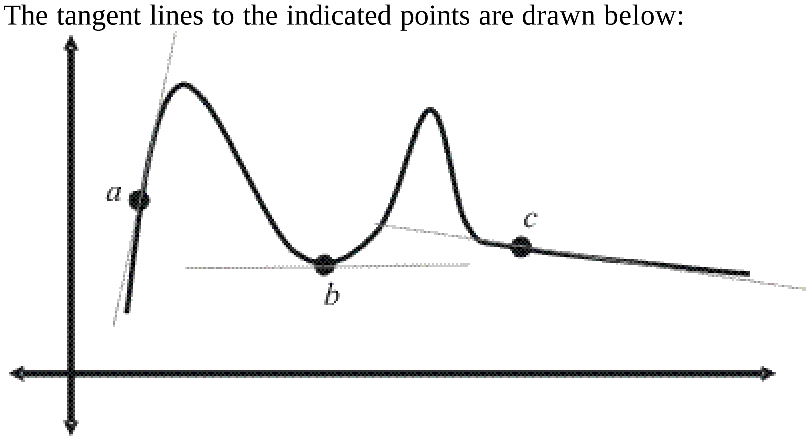
$$m_{\tan} = \lim_{h\to 0}\frac{g(16+h) - g(16)}{h} = \lim_{h\to 0}\frac{\left(0.45(16+h)^2 - 10\right) - \left(0.45(16)^2 - 10\right)}{h}$$
$$= \lim_{h\to 0}\frac{0.45\left(256 + 32h + h^2\right) - 10 - 115.2 + 10}{h} = \lim_{h\to 0}\frac{h(14.4 + 0.45h)}{h}$$
$$= \lim_{h\to 0}\left(14.4 + 0.45h\right) = 14.4$$

Applet Method Using the applet to find the approximate slope of the tangent line is 14.4.



Question 4.10

The tangent lines to the indicated points are drawn below:

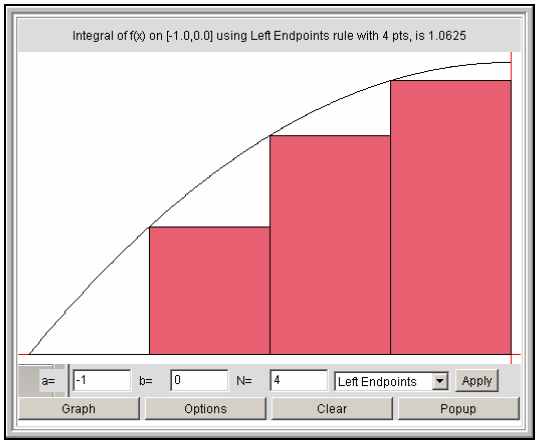


At *a*, the tangent line has positive slope, at *b* the tangent line has zero slope and at *c* the tangent line has negative slope.

Chapter 9 – Definite Integrals

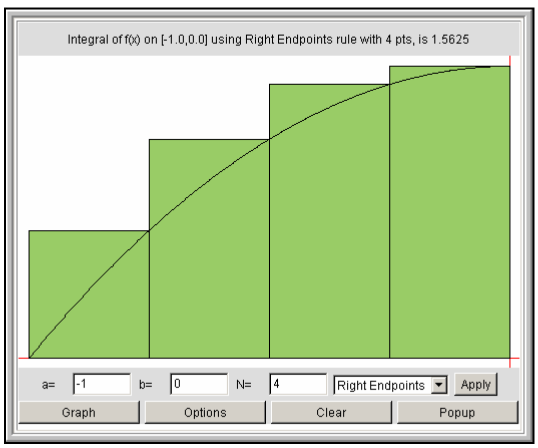
Question 9.1

Left Endpoint Method using the Integration Applet



The area at the top of the applet is 1.0625. This is an underestimate since each rectangle lies below the curve.

Right Endpoint Method using the Integration Applet



The area at the top of the applet is 1.5625. This is an overestimate since the rectangles extend past the function and capture too much area.

Question 9.2

We are looking at the area under the curve from x = -2 to x = 2. Since the entire area is above the x- axis we have

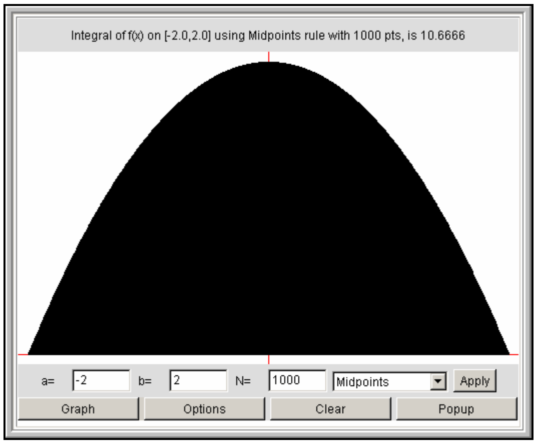
$$\int_{-2}^{2}\left(\frac{8}{x^2+1}\right)dx$$

Question 9.3

Algebraic Method: Using the Fundamental Theorem of Calculus, we have

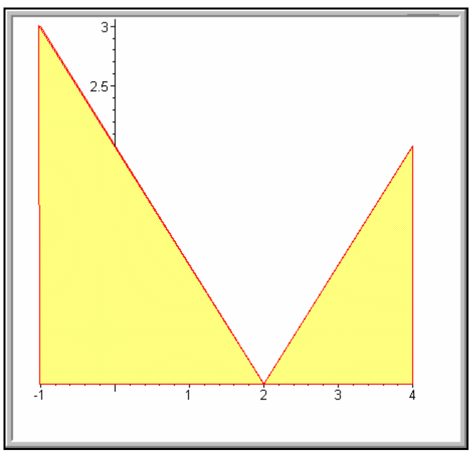
$$\int_{-2}^{2}\left(-x^2+4\right)dx = \left.\frac{-x^3}{3}+4x\right]_{-2}^{2} = \left(\frac{-2^3}{3}+4(2)\right)-\left(\frac{-(-2)^3}{3}+4(-2)\right)=\frac{32}{3}$$

Applet Method: Using the Area Between Two Curves Applet, we can approximate the integral by finding the area between the functions $y=-x^2+4$ and $y=0$. To get the best estimate we will use a large number of rectangles (1000) and the midpoints, as shown below.



Question 9.4

The graph of $f(x)=|x-2|$ with the area equivalent to the integral asked is given by



Geometric Area Method:

$$\int_{-1}^{4}|x-2|\,dx = Area_{R1}+Area_{R2} = 0.5(3)(3)+0.5(2)(2)=6.5$$

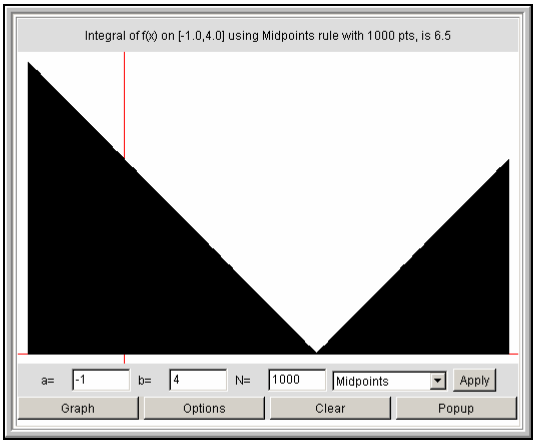
Algebraic Method: Using the Fundamental Theorem of Calculus, we have

$$\int_{-1}^{4}|x-2|\,dx = \int_{-1}^{2}(-x+2)dx+\int_{2}^{4}(x-2)dx = \left.\frac{-x^2}{2}+2x\right]_{-1}^{2}+\left.\frac{x^2}{2}-2x\right]_{2}^{4}$$

$$=\left[\left(\tfrac{-2^2}{2}+2(2)\right)-\left(\tfrac{-(-1)^2}{2}+2(-1)\right)\right]+\left[\left(\tfrac{4^2}{2}-2(4)\right)-\left(\tfrac{2^2}{2}-2(2)\right)\right]$$

$$=6.5$$

Applet Method: Using the Area Between Two Curves Applet, we can approximate the integral by finding the area between the functions $y = |x - 2|$ and $y = 0$. To get the best estimate we will use a large number of rectangles (1000) and the Midpoints, as shown below.



Question 9.5

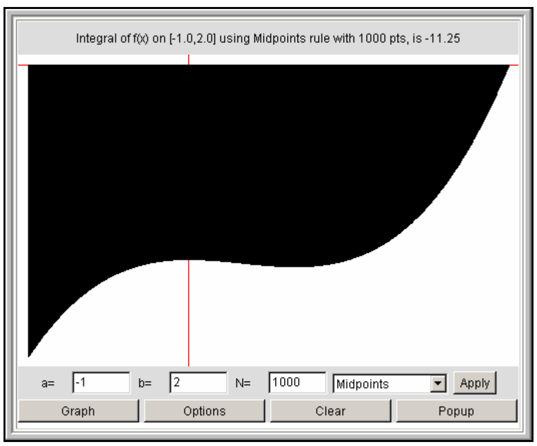
Algebraic Method: Using the Fundamental Theorem of Calculus, we have

$$\int_{-1}^{2}(x^3 - x^2 - 4)dx = \frac{x^4}{4} - \frac{x^3}{3} - 4x\Bigg]_{=-1}^{2}$$

$$= \left(\frac{2^4}{4} - \frac{2^3}{3} - 4(2)\right) - \left(\frac{(-1)^4}{4} - \frac{(-1)^3}{3} - 4(-1)\right)$$

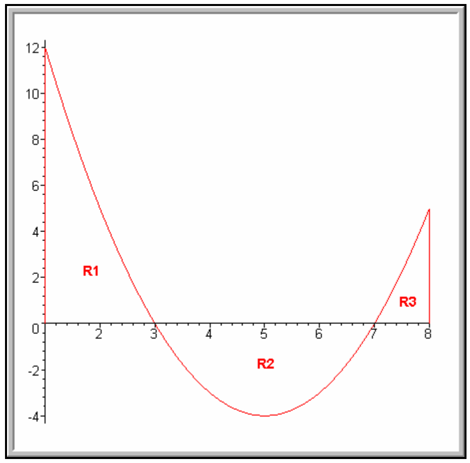
$$= -11.25$$

Applet Method: Using the Area Between Two Curves Applet, we can approximate the integral by finding the area between the functions $y = x^3 - x^2 - 4$ and $y = 0$. To get the best estimate we will use a large number of rectangles (1000) and the Midpoints, as shown below.



Question 9.6

Graphing the function $x^2 - 10x + 21$ on the interval [1, 8] gives



Algebraic Method: Using the Fundamental Theorem of Calculus, we have

$$R_1 = \int_1^3 (x^2 - 10x + 21)dx = \left.\frac{x^3}{3} - 5x^2 + 21x\right]_1^3 = \left(\frac{3^3}{3} - 5(3)^2 + 21(3)\right) - \left(\frac{1^3}{3} - 5(1)^2 + 21(1)\right) = \frac{32}{3}$$

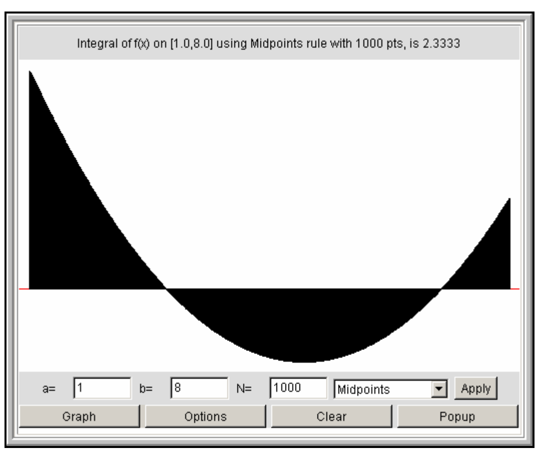
$$R_2 = \int_3^7 (x^2 - 10x + 21)dx = \left.\frac{x^3}{3} - 5x^2 + 21x\right]_3^7 = \left(\frac{7^3}{3} - 5(7)^2 + 21(7)\right) - \left(\frac{3^3}{3} - 5(3)^2 + 21(3)\right) = -\frac{32}{3}$$

$$R_3 = \int_7^8 (x^2 - 10x + 21)dx = \left.\frac{x^3}{3} - 5x^2 + 21x\right]_7^8 = \left(\frac{8^3}{3} - 5(8)^2 + 21(8)\right) - \left(\frac{7^3}{3} - 5(7)^2 + 21(7)\right) = \frac{7}{3}$$

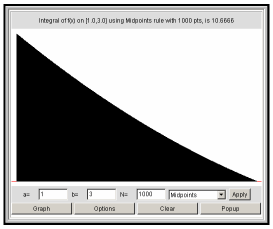
Net Area = $R_1 + R_2 + R_3 = \frac{32}{3} - \frac{32}{3} + \frac{7}{3} = \frac{7}{3}$

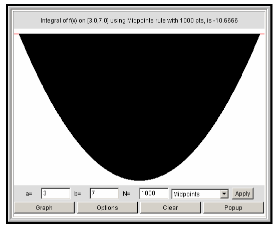
Gross Area = $|R_1| + |R_2| + |R_3| = \frac{32}{3} + \frac{32}{3} + \frac{7}{3} = \frac{71}{3}$

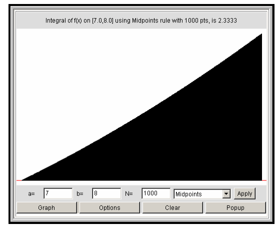
Applet Method: Using the Area Between Two Curves Applet, we can approximate the net area by finding the area between the functions $y = x^2 - 10x + 21$ and $y = 0$. To get the best estimate we will use a large number of rectangles (1000) and the Midpoints, as shown below.



Using the Area Between Two Curves Applet, we can approximate the gross area by finding the absolute value of the integral between the functions $y = x^2 - 10x + 21$ and $y = 0$ in each of the three regions and adding the values. To get the best estimate we will use a large number of rectangles (1000) and the Midpoints on each interval, as shown below.



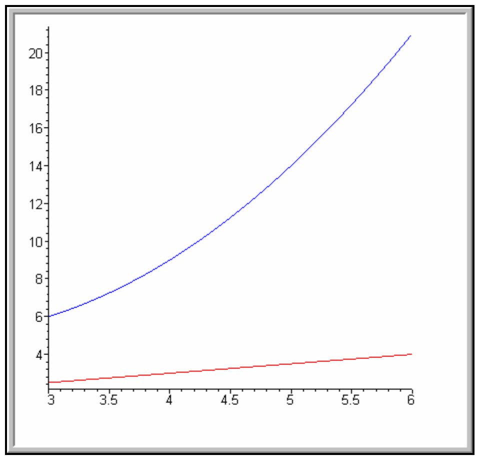




Thus, we have 10.6666 + 10.6666 + 2.3333 = 23.6665 as a close approximation to the gross area.

Question 9.7

Graphing both $f(x) = 0.5x + 1$ and $g(x) = x^2 - 4x + 9$ on the given interval, [3, 6], we have



<u>Algebraic Method:</u> The area between the curves is given by

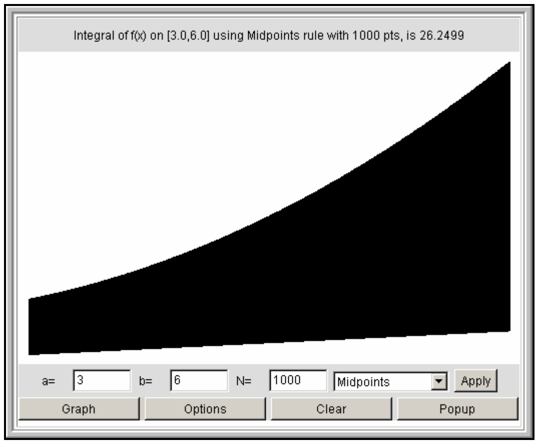
$$\int_3^6 (Top - Bottom)\,dx = \int_3^6 \left[\left(x^2 - 4x + 9\right) - \left(0.5x + 1\right)\right] dx = \int_3^6 \left(x^2 - 4.5x + 8\right) dx$$

Using the Fundamental Theorem of Calculus, we have

$$\int_3^6 \left(x^2 - 4.5x + 8\right) dx = \tfrac{x^3}{3} - \tfrac{9}{4}x^2 + 8x\Big]_3^6 = \left(\tfrac{6^3}{3} - \tfrac{9}{4}(6)^2 + 8(6)\right) - \left(\tfrac{3^3}{3} - \tfrac{9}{4}(3)^2 + 8(3)\right)$$

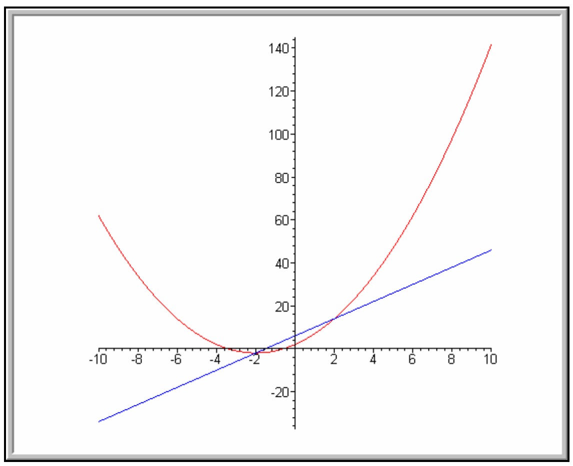
$$= 26.25$$

Applet Method: Using the Area Between Two Curves Applet, we can approximate the integral. Make sure you enter the functions as: "top curve, bottom curve". To get the best estimate we will use a large number of rectangles (1000) and the Midpoints, as shown below.



Question 9.8

Graphing both $f(x) = x^2 + 4x + 2$ and $g(x) = 4x + 6$, we have the area bounded by the two curves as



We can find the two points of intersection by setting the two equations equal to one another, as follows:

$$x^2 + 4x + 2 = 4x + 6$$
$$x^2 + 4x + 2 - 4x - 6 = 0$$
$$x^2 - 4 = 0$$
$$x^2 = 4$$
$$x = \pm 2$$

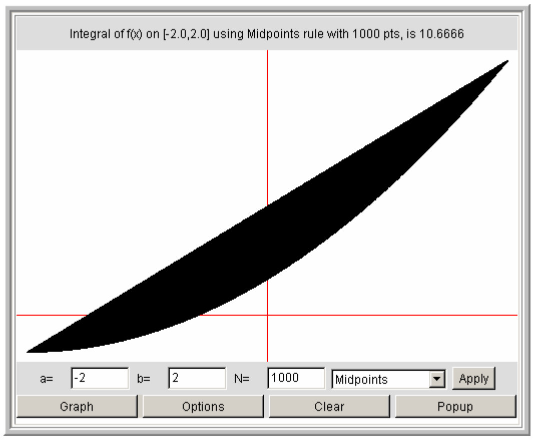
So, we have the area is bounded between *x* = -2 and *x* = 2. Therefore, the area between the curves is given by

$$\int_{-2}^{2} (Top - Bottom)\,dx = \int_{-2}^{2} \left[(4x+6) - (x^2 + 4x + 2) \right] dx = \int_{-2}^{2} \left(-x^2 + 4 \right) dx \,.$$

Algebraic Method: Using the Fundamental Theorem of Calculus, we have

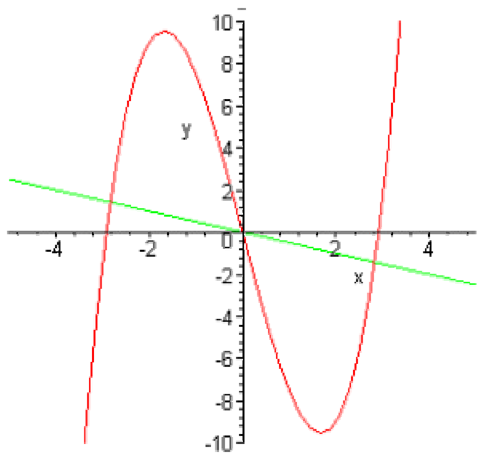
$$\int_{-2}^{2}\left(-x^2+4\right)dx = \frac{-x^3}{3}+4x\Big]_{-2}^{2} = \left(\frac{-(2)^3}{3}+4(2)\right)-\left(\frac{-(-2)^3}{3}+4(-2)\right)=\frac{32}{3}$$

Applet Method



Question 9.9

Graphing both $f(x)=x^3-8.5x$ and $g(x)=0.5x$, we have the area bounded by the two curves as



Finding the three points of intersection we have two areas - one bounded between $x=-3$ and $x = 0$, and another between $x = 0$ and $x=3$. Therefore, the area between the curves is given by

$$\int_{-3}^{0}(Top-Bottom)\,dx+\int_{0}^{3}(Top-Bottom)\,dx=\int_{-3}^{0}\left[\left(x^3-8.5x\right)-\left(0.5x\right)\right]dx+\int_{0}^{3}\left[\left(0.5x\right)-\left(x^3-8.5x\right)\right]dx$$

$$=\int_{-3}^{0}\left(x^3-9x\right)dx+\int_{0}^{3}\left(-x^3+9x\right)dx$$

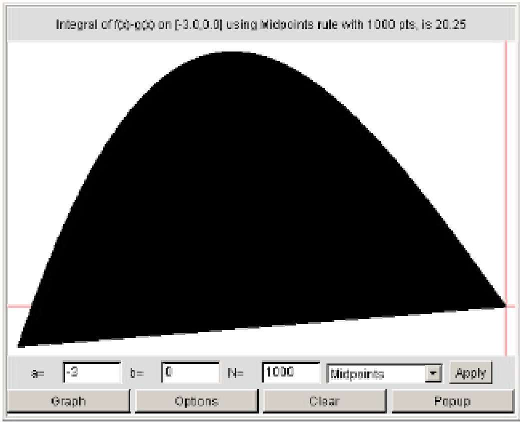
Algebraic Method: Using the Fundamental Theorem of Calculus, we have

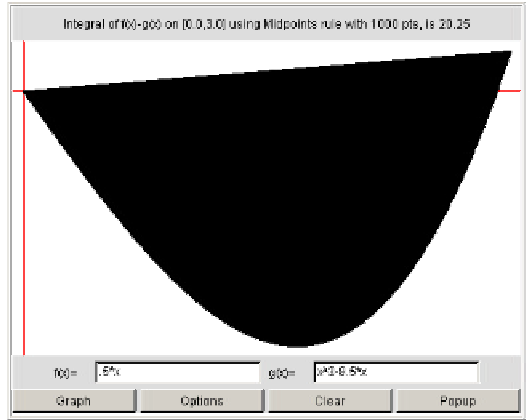
$$\int_{-3}^{0}\left(x^3-9x\right)dx+\int_{0}^{3}\left(-x^3+9x\right)dx=\frac{x^4}{4}-4.5x^2\Big]_{-3}^{0}+\frac{-x^4}{4}+4.5x^2\Big]_{0}^{3}$$

$$=\left[\left(\frac{0^4}{4}-4.5(0)^2\right)-\left(\frac{(-3)^4}{4}-4.5(-3)^2\right)\right]+\left[\left(\frac{-(3)^4}{4}+4.5(3)^2\right)-\left(\frac{-(0)^4}{4}+4.5(0)^2\right)\right]$$

$$=20.25+20.25=40.5$$

Applet Method: Using the Area-Between-Two-Curves Applet, we can approximate the area by finding the area of each region and adding the areas together. Make sure you enter the top function as $f(x)$ and the bottom function as $g(x)$ on each region. To get the best estimate we will use a large number of rectangles (1000) and the Midpoints, as shown below.

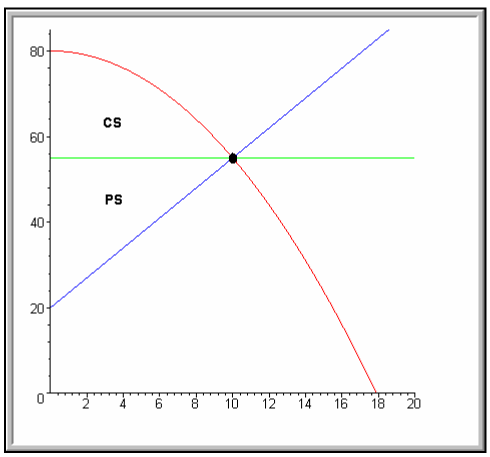




Thus, an approximation of the area between the curves is 2(20.25) = 40.5.

Question 9.10

First, graph both the supply and demand functions and locate the equilibrium point.



Next, calculate the equilibrium quantity by setting supply equal to demand, as follows:

$$-0.25x^2 + 80 = 3.5x + 20$$

$$0 = -0.25x^2 + 80 - 3.5x - 20 = -0.25x^2 - 3.5x + 60$$

$$= -0.25(x^2 + 14x - 240) = -0.25(x + 24)(x - 10)$$

Therefore, the equilibrium quantity is at $x = 10$. Substitute this value into either the supply or demand equation to find the corresponding equilibrium price:

$$S(10) = 3.5(10) + 20 = 55$$

So, the equilibrium point is at (10, 55).

Consumer Surplus (CS):

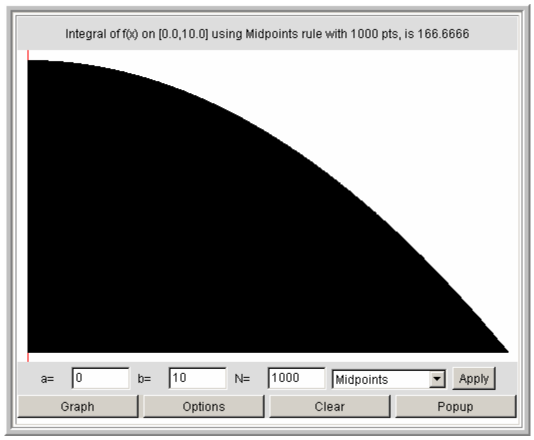
Algebraic Method

$$\text{CS} = \int_0^{x_0} (D(x) - p_0)\,dx = \int_0^{10} (-0.25x^2 + 80 - 55)\,dx = \int_0^{10} (-0.25x^2 + 25)\,dx$$

$$= -0.25\left(\frac{x^3}{3}\right) + 25x\Big]_0^{10} = \left(\tfrac{-1}{12}(10)^3 + 25(10)\right) - \left(\tfrac{-1}{12}(0)^3 + 25(0)\right) = \frac{500}{3}$$

Since this value is in hundreds of dollar, the consumer surplus is $500/3 \times 100 = 16{,}666.67$ or about \$16,667.

Applet Method: Using the Area Between Two Curves Applet, we can approximate the integral. Make sure you enter the functions as: $f(x)$ =demand curve and $g(x)$ =equilibrium price". To get the best estimate we will use a large number of rectangles (1000) and the Midpoints, as shown below.



Producer Surplus (PS):

Algebraic Method

$$\text{PS} = \int_0^{x_0} (p_0 - S(x))\,dx = \int_0^{10} (55 - (3.5x + 20))\,dx = \int_0^{10} (35 - 3.5x)\,dx$$

$$= 35x - 3.5\left(\frac{x^2}{2}\right)\Big]_0^{10} = \left(35(10) - 1.75(10)^2\right) - \left(35(0) - 1.75(0)^2\right) = 175$$

Applet Method: Using the Area Between Two Curves Applet, we can approximate the integral. Make sure you enter the functions as: $f(x)$ =equilibrium price, $g(x)$ =supply curve. To get the best estimate we will use a large number of rectangles (1000) and the Midpoints, as shown below.