

Classification of Strictly Weakly Integral Modular Categories of Dimension $16p$

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Abstract

A modular category is a non-degenerate braided fusion category equipped with a ribbon structure. A complete classification of modular categories is motivated by applications to physics, where unitary modular categories correspond to $(2+1)$ -dimensional topological quantum field theories. Here, we classify strictly weakly integral modular categories of dimension $16p$.

1 Introduction

The 2016 Nobel Prize in Physics was awarded to Thouless, Haldane, and Kosterlitz for “theoretical discoveries of topological phase transitions and topological phases of matter.” $(2+1)$ -dimensional topological quantum field theories can arise from unitary modular categories [KSW02]. Because of their widespread applications in theoretical physics as well as pure mathematics, it is desirable to have a complete classification of modular categories.

There exist both rank-based and dimension-based approaches to classification. It was shown in [BNRW13] that there are finitely many modular categories of a given rank, and [BNRW13, BGN⁺14, BR10, RSW07] provide complete classification through rank 5 and partial classification through rank 11. We take a dimension-based approach to classification previously studied in [BGH⁺13, DT14, BGN⁺14, BPR16]. Integral modular categories of dimension pq^4 were classified in [BGH⁺13, DT14]. We weaken the condition of integrality and characterize strictly weakly integral modular categories of dimension $16p$, where p is an odd prime. Strictly weakly integral modular categories are necessarily of even dimension, and in particular must have dimension divisible by four [BPR16]. Modular categories of dimension $4p$ were classified in [BGN⁺14] and $8p$ in [BPR16].

2 Preliminaries

A modular category is a non-degenerate braided fusion category equipped with a ribbon structure. Fusion categories are semisimple, and in particular all objects are direct sums of simple objects. Fusion categories also have finite rank, the number of isomorphism classes of simple objects. Each simple object X has a Frobenius-Perron dimension given by the largest nonnegative eigenvalue of the matrix N_X of left-multiplication by X . The Frobenius-Perron dimension of a fusion category \mathcal{C} is $\text{FPDim}(\mathcal{C}) = \sum \text{FPDim}(X_i)^2$ summed over all isomorphism classes of simple objects $X_i \in \mathcal{C}$. A fusion category \mathcal{C} is weakly integral if $\text{FPDim}(\mathcal{C}) \in \mathbb{Z}$ and integral if $\text{FPDim}(X_i) \in \mathbb{Z}$ for all simple $X_i \in \mathcal{C}$. A fusion category is pointed if all of its simple objects are invertible. If \mathcal{C} is weakly integral, then $\text{FPDim}(X_i)^2 \mid \text{FPDim}(\mathcal{C})$ for all simple objects $X_i \in \mathcal{C}$, and it follows that all simple objects $X_i \in \mathcal{C}$ have $\text{FPDim}(X_i) = \sqrt{n_i}$ for some $n_i \in \mathbb{Z}^+$. \mathcal{C} is strictly weakly integral if it is weakly integral with at least one simple object of non-integral dimension. If \mathcal{C} is modular and strictly weakly integral, then $4 \mid \text{FPDim}(\mathcal{C})$ [BPR16].

A fusion category \mathcal{C} is braided if there is a family of natural isomorphisms $C_{X,Y} : X \otimes Y \rightarrow Y \otimes X$ satisfying the hexagon axioms. In particular, the associative tensor product is also abelian. The Müger center of a braided fusion category \mathcal{C} is a fusion subcategory defined

$$Z_2(\mathcal{C}) = \{X \in \mathcal{C} : C_{Y,X} \circ C_{X,Y} = \text{id}_{X \otimes Y} \ \forall Y \in \mathcal{C}\}$$

A braided fusion category is non-degenerate if its Müger center is the trivial fusion subcategory generated by the unit object, and symmetric if $Z_2(\mathcal{C}) = \mathcal{C}$. For braided fusion categories $\mathcal{D} \subseteq \mathcal{C}$, the relative center or centralizer of \mathcal{D} in \mathcal{C} is denoted

$$\mathcal{D}' = Z_{\mathcal{C}}(\mathcal{D}) = \{X \in \mathcal{C} : C_{Y,X} \circ C_{X,Y} = \text{id}_{X \otimes Y} \ \forall Y \in \mathcal{D}\}$$

If \mathcal{C} is nondegenerate then $(\mathcal{D}')' = \mathcal{D}$ and $\text{FPDim}(\mathcal{D}) \cdot \text{FPDim}(\mathcal{D}') = \text{FPDim}(\mathcal{C})$. If $\mathcal{D} \subseteq \mathcal{C}$ are both modular, then \mathcal{D}' is also modular and $\mathcal{C} \cong \mathcal{D} \boxtimes \mathcal{D}'$. If \mathcal{C} is modular, then $\mathcal{C}_{pt} = \mathcal{C}'_{ad}$ [BGH⁺13].

A fusion category \mathcal{C} is graded by a group G if \mathcal{C} can be decomposed into a direct sum of full abelian subcategories indexed by G , $\mathcal{C} = \bigoplus_{g \in G} \mathcal{C}_g$ such that $\mathcal{C}_g \otimes \mathcal{C}_h \subset \mathcal{C}_{gh}$ for all $g, h \in G$. A grading is called faithful if all \mathcal{C}_g are nonempty. In a faithful grading, all components have the same dimension, $\text{FPDim}(\mathcal{C}_g) = \frac{\text{FPDim}(\mathcal{C})}{|G|}$.

Every fusion category is faithfully graded by its universal grading group, $\mathcal{U}(\mathcal{C})$, and every faithful grading group is a quotient of $\mathcal{U}(\mathcal{C})$ [GN06]. In a modular category, $\mathcal{U}(\mathcal{C}) \cong \mathcal{G}(\mathcal{C})$, the group of isomorphism classes of invertible simple objects. The trivial component of the universal grading is $\mathcal{C}_e = \mathcal{C}_{ad}$, the fusion subcategory generated by subobjects of $X \otimes X^*$ where X runs through all simple objects of \mathcal{C} .

A strictly weakly integral fusion category is faithfully graded by an elementary abelian 2-group E , with $|E| = 2^k$ for some k [GN06]. This grading is called the dimensional grading or the GN-grading, and corresponds to partitioning the simple objects by dimension. For each $g \in E$, there is a distinct square-free positive integer n_g with $n_e = 1$ and $\text{FPDim}(X) \in \sqrt{n_g}\mathbb{Z}$ for all simple $X \in \mathcal{C}_g$. Thus the identity component is the fusion subcategory generated by the simple objects of integer dimension, $\mathcal{C}_e = \mathcal{C}_{int}$.

3 Results

Let \mathcal{C} be a strictly weakly integral modular category with $\text{FPDim}(\mathcal{C}) = 16p$, for p an odd prime. We have $\text{FPDim}(X_i) \in \{1, 2, 4, \sqrt{2}, 2\sqrt{2}, \sqrt{p}, 2\sqrt{p}, 4\sqrt{p}, \sqrt{2p}, 2\sqrt{2p}\}$ for all simple X_i . Thus the dimensions of simple objects can be partitioned into \mathbb{Z} , $\sqrt{2}\mathbb{Z}$, $\sqrt{p}\mathbb{Z}$, and $\sqrt{2p}\mathbb{Z}$, and $|E| \in \{2, 4\}$.

We will use the following variables to denote the number of simple objects of each dimension:

FPDim	1	2	4	$\sqrt{2}$	$2\sqrt{2}$	\sqrt{p}	$2\sqrt{p}$	$4\sqrt{p}$	$\sqrt{2p}$	$2\sqrt{2p}$
# simples	a	b	c	f	d	h	k	l	m	n

In the dimensional grading, each component has equal dimension, and in particular $\text{FPDim}(\mathcal{C}_{int}) = \frac{\text{FPDim}(\mathcal{C})}{|E|}$. In the integral component, we must have $\text{FPDim}(\mathcal{C}_{int}) = a + 4b + 16c$. We have $a = \text{FPDim}(\mathcal{C}_{pt}) = |\mathcal{U}(\mathcal{C})|$. We must have $\text{FPDim}(\mathcal{C}_{pt}) | \text{FPDim}(\mathcal{C}_{int})$ because the pointed subcategory is a fusion subcategory of the integral subcategory, and $|E| | |\mathcal{U}(\mathcal{C})|$ because E is a quotient of $\mathcal{U}(\mathcal{C})$.

Lemma 3.1. *If \mathcal{C} is a strictly weakly integral modular category with $\text{FPDim}(\mathcal{C}) = 2^n p$ and $p | \text{FPDim}(\mathcal{C}_{pt})$, then $|E| = 2$.*

Proof. The components of the universal grading must have dimension 2^k for some $k \geq 1$. Then the components of the universal grading cannot accommodate simple objects with dimensions in $\sqrt{p}\mathbb{Z}$ or $\sqrt{2p}\mathbb{Z}$. So all non-integral simple objects must have dimensions in $\sqrt{2}\mathbb{Z}$, so $|E| = 2$. □

From the above, we obtain five possible cases. When $|E| = 2$, $a \in \{4, 8, 4p, 8p\}$. When $|E| = 4$, $a = 4$.

3.1 $|E| = 2$, $a = 8p$

Definition 3.1. A generalized Tambara-Yamagami category is a non-pointed fusion category in which the tensor product of two non-invertible simple objects is a direct sum of invertible objects.

Lemma 3.2. *If \mathcal{C} is a strictly weakly integral modular category with $\text{FPDim}(\mathcal{C}) = 2^n p$ and $\text{FPDim}(\mathcal{C}_{pt}) = 2^{n-1} p$, then \mathcal{C} is a Deligne product of an Ising category and a pointed modular category.*

Proof. The components of the universal grading must have dimension 2, so they can only accommodate non-integral simple objects of dimension $\sqrt{2}$. In the dimensional grading, $\mathcal{C} = \mathcal{C}_0 \oplus \mathcal{C}_1$ with $\mathcal{C}_0 = \mathcal{C}_{int} = \mathcal{C}_{pt}$. Any tensor product of non-integral objects is a direct sum of invertibles, $\mathcal{C}_1 \otimes \mathcal{C}_1 \subset \mathcal{C}_0 = \mathcal{C}_{pt}$. Thus \mathcal{C} is a generalized Tambara-Yamagami category. By [BPR16], any modular generalized Tambara-Yamagami category is a Deligne product of an Ising category and a pointed modular category. \square

By the above lemma, \mathcal{C} is a Deligne product of an Ising category and a pointed modular category.

3.2 $|E| = 2$, $a = 8$

The components of the universal grading have dimension $2p$. For the integral components of the universal grading, we have $2p = a_g + 4b_g + 16c_g$ and thus $a_g \equiv 2 \pmod{4}$. Thus $a_g \geq 2$ over 4 integral components, so we must have $a_g = 2$ in each component. Each integral component also contains some non-invertible simple objects, with $b_g \equiv \frac{p-1}{2} \pmod{4}$ and $c_g = \frac{\frac{p-1}{2} - b_g}{4}$.

In the non-integral components of the universal grading, we have either $f_g \equiv p \pmod{4}$ and $d_g = \frac{p-f_g}{4}$, $h_g = 2$, or $m_g = 1$.

Lemma 3.3. *If $\mathcal{C}_{ad} \neq \mathcal{C}_{int}$, then $\mathcal{Z}_2(\mathcal{C}_{int}) = \mathcal{C}'_{int} \subsetneq \mathcal{C}_{pt}$.*

Proof. $\mathcal{C}_{ad} \subseteq \mathcal{C}_{int}$ and thus $\mathcal{C}'_{int} \subseteq \mathcal{C}'_{ad} = \mathcal{C}_{pt}$. We have $\mathcal{C}'_{int} \subseteq \mathcal{C}_{pt} \subseteq \mathcal{C}_{int}$, so $\mathcal{Z}_2(\mathcal{C}_{int}) = \mathcal{C}'_{int}$. Now suppose $\mathcal{C}'_{int} = \mathcal{C}_{pt}$. Then $\mathcal{C}''_{int} = \mathcal{C}_{int} = \mathcal{C}_{ad}$, a contradiction. Thus $\mathcal{C}'_{int} \subsetneq \mathcal{C}_{pt}$. \square

Lemma 3.4. *If $\mathcal{C}_{int} \neq \mathcal{C}_{pt}$, then $\mathcal{Z}_2(\mathcal{C}_{int}) = \mathcal{C}'_{int} \subsetneq \mathcal{C}_{ad}$.*

Proof. $\mathcal{C}_{pt} \subseteq \mathcal{C}_{int}$ and thus $\mathcal{C}'_{int} \subseteq \mathcal{C}'_{pt} = \mathcal{C}_{ad} \subseteq \mathcal{C}_{int}$, so $\mathcal{Z}_2(\mathcal{C}_{int}) = \mathcal{C}'_{int}$. Now suppose $\mathcal{C}'_{int} = \mathcal{C}_{ad}$. Then $\mathcal{C}''_{int} = \mathcal{C}_{int} = \mathcal{C}_{pt}$, a contradiction. So $\mathcal{C}'_{int} \subsetneq \mathcal{C}_{ad}$. \square

Lemma 3.5. *If \mathcal{C} is a strictly weakly integral modular category and $|\mathcal{C}_{int}| = \frac{|\mathcal{C}|}{2}$, then \mathcal{C}_{int} is not modular.*

Proof. If \mathcal{C}_{int} is modular, then $\mathcal{C} \cong \mathcal{C}_{int} \boxtimes \mathcal{D}$ for some $|\mathcal{D}| = 2$. Then $4 \nmid |\mathcal{D}|$, so it must be integral. But then \mathcal{C} must be integral, a contradiction. \square

So $\mathcal{Z}_2(\mathcal{C}_{int}) \subseteq (\mathcal{C}_{ad})_{pt}$ as fusion subcategories, and \mathcal{C}_{int} is not modular. $(\mathcal{C}_{ad})_{pt} = \{\mathbb{1}, g\} = \langle g \rangle$ where g is the unique nontrivial, self-dual invertible in \mathcal{C}_{ad} . So $\mathcal{Z}_2(\mathcal{C}_{int}) = \langle g \rangle$. g fixes all simple objects with dimensions in $2\mathbb{Z}$, $\sqrt{2}\mathbb{Z}$, and $\sqrt{2p}\mathbb{Z}$, and maps each simple object with dimension 1 or dimension in $\sqrt{p}\mathbb{Z}$ to the other simple object of the same dimension in the same component of the universal grading.

$\langle g \rangle$ is symmetric, so it is either Tannakian (g a boson) or $s\text{Vec}$ (g a fermion) [BPR16].

Lemma 3.6 ([Mue98], Lemma 5.4). *If $\langle g \rangle = s\text{Vec} \subseteq \mathcal{D}'$ for some premodular category \mathcal{D} , then $g \otimes Y \not\cong Y$ for all simple $Y \in \mathcal{D}$.*

Case i: $\langle g \rangle = \text{sVec}$

Then $\text{sVec} \subseteq \mathcal{C}_{pt} = \mathcal{C}'_{ad}$, so we must have $g \otimes Y \cong Y$ for all simple $Y \in \mathcal{C}_{ad}$. But g stabilizes all of the non-invertible simple objects in \mathcal{C}_{ad} , and there must be at least one such object. So we cannot have $\langle g \rangle = \text{sVec}$.

Case ii: $\langle g \rangle$ Tannakian

We have $\mathcal{C} \supset \langle g \rangle = \text{Rep}(\mathbb{Z}_2)$ and we can consider $\mathcal{C}_{\mathbb{Z}_2}$, the \mathbb{Z}_2 -de-equivariantization of \mathcal{C} . $\text{FPDim}(\mathcal{C}_{\mathbb{Z}_2}) = \frac{\text{FPDim}(\mathcal{C})}{|\mathbb{Z}_2|} = 8p$, so it is weakly integral. This case is only possible when the non-integral simple objects have dimension \sqrt{p} . Simple objects with dimension $\sqrt{2}$ or $\sqrt{2p}$ are stabilized by g , so $\mathcal{C}_{\mathbb{Z}_2}$ would contain simple objects with dimension $\frac{\sqrt{2}}{2}$ or $\frac{\sqrt{2p}}{2}$, which contradicts that $\mathcal{C}_{\mathbb{Z}_2}$ is weakly integral.

We use unprimed variables to denote the total number of simple objects of each dimension in \mathcal{C} , and primed variables to denote the total numbers in $\mathcal{C}_{\mathbb{Z}_2}$. In $\mathcal{C}_{\mathbb{Z}_2}$, we have $a' = 4 + 2b$, $b' = 2c$ and $h' = 4$. In the dimensional grading, we have $|E| = 2$ and $|(\mathcal{C}_{\mathbb{Z}_2})_{int}| = 4p = 4 + 2b + 8c$. Then $2 + b + 4c = 2p \equiv 2 \pmod{4}$ and so $4|b$. We must have $a' = |(\mathcal{C}_{\mathbb{Z}_2})_{pt}| = 4 + 2b = 4(1 + \frac{b}{2})|4p = |(\mathcal{C}_{\mathbb{Z}_2})_{int}|$. Then $\frac{b}{2} \in \{0, p-1\}$. Then $(b, c) \in \{(0, \frac{p-1}{2}), (2p-2, 0)\}$. But $b = 0$ is impossible because $\text{FPDim}(X \otimes X^*) = 16$ when $\text{FPDim}(X) = 4$, but there are only two invertibles in \mathcal{C}_{ad} . So $c = 0$ and $\mathcal{C}_{\mathbb{Z}_2}$ is a generalized Tambara-Yamagami category. $(\mathcal{C}_{int})_{\mathbb{Z}_2}$ is modular because $\mathcal{Z}_2(\mathcal{C}_{int}) = \text{Rep}(\mathbb{Z}_2)$. So \mathcal{C} is a gauging of a pointed modular category of dimension $4p$.

3.3 $|E| = 2$, $a = 4p$

In the universal grading, we have p components with $a_g = 4$, p components with $b_g = 1$, and $2p$ components with $f_g = 2$. \mathcal{C}_{ad} is pointed and thus \mathcal{C} is nilpotent.

So $\mathcal{Z}_2(\mathcal{C}_{int}) \subsetneq \mathcal{C}_{ad}$ as fusion subcategories and \mathcal{C}_{int} cannot be modular, so $\mathcal{Z}_2(\mathcal{C}_{int}) = \langle g \rangle$ for some nontrivial self-dual invertible $g \in \mathcal{C}_{ad}$. $\langle g \rangle$ is symmetric, so it is either Tannakian or sVec .

Case i: $\langle g \rangle = \text{sVec}$

We have $\langle g \rangle = \text{sVec} = \mathcal{C}'_{int}$, so g cannot stabilize any simple objects in \mathcal{C}_{int} . But all simple objects of dimension 2 are stabilized by all of \mathcal{C}_{ad} . So this case is impossible.

Case ii: $\langle g \rangle = \text{Rep}(\mathbb{Z}_2)$

$\mathcal{C}_{\mathbb{Z}_2}$ is weakly integral, so g cannot stabilize any simple objects of dimension $\sqrt{2}$. So we must have $f' = \frac{f}{2} = 2p$. g stabilizes all simple objects of dimension 2, and g cannot stabilize any invertibles. So $a' = \frac{a}{2} + 2b = 4p$ and $(\mathcal{C}_{int})_{\mathbb{Z}_2}$ is pointed. So $\mathcal{C}_{\mathbb{Z}_2}$ is a generalized Tambara-Yamagami category. $(\mathcal{C}_{int})_{\mathbb{Z}_2}$ is modular because $\mathcal{Z}_2(\mathcal{C}_{int}) = \text{Rep}(\mathbb{Z}_2)$. So \mathcal{C} is a gauging of a pointed modular category of dimension $4p$.

Remark 3.1. We have $g \in \mathcal{C}_{ad}$ is a nontrivial self-dual invertible that does not stabilize any simple X with $\text{FPDim}(X) = \sqrt{2}$. But every such X is stabilized by exactly one nontrivial self-dual invertible in \mathcal{C}_{ad} . So all of the nontrivial invertibles g_i in \mathcal{C}_{ad} are self-dual.

3.4 $|E| = 4$, $a = 4$

In this case $E = \mathcal{U}(\mathcal{C}) = \mathbb{Z}_2 \times \mathbb{Z}_2 \cong \mathcal{G}(\mathcal{C})$. Thus all of the invertibles are self-dual. We have $\mathcal{C}_{int} = \mathcal{C}_{ad}$ and thus $\mathcal{C}'_{int} = \mathcal{C}_{pt} \subset \mathcal{C}_{int}$, so $\mathcal{C}'_{int} = \mathcal{Z}_2(\mathcal{C}_{int}) = \mathcal{C}_{pt} \cong \mathbb{Z}_2 \times \mathbb{Z}_2$. So \mathcal{C}_{pt} is symmetric, so $\langle g \rangle$ is symmetric for all $g \in \mathcal{C}_{pt}$.

In the integral component, we have $a = 4$, $b \equiv p-1 \pmod{4}$, and $c = \frac{p-1-b}{4}$. In the non-integral components we have $f \equiv 2 \pmod{4}$ and $d = \frac{2p-f}{4}$, $h = 4$ or $k = 1$, and $m = 2$.

\mathcal{C}_{pt} has three nontrivial invertibles, g, h, gh , which are either bosons ($\theta=1$) or fermions ($\theta = -1$). We must have $\theta_g\theta_h = \theta_{gh}$, so there are either two fermions or no fermions.

3.5 $|E| = 2, a = 4$

In the trivial component of the universal grading, we have $a_g = 4, b_g \equiv_4 p - 1$, and $c_g = \frac{p-1-b_g}{4}$. In the nontrivial integral component, we have $b_g \equiv_4 p$ and $c_g = \frac{p-b_g}{4}$. In the non-integral components, we have either $f_g \equiv_4 2$ and $d_g = \frac{2p-f_g}{4}, h_g = 4$ or $k_g = 1$, or $m_g = 2$.

We have $\mathcal{Z}_2(\mathcal{C}_{int}) \subsetneq \mathcal{C}_{pt}$ as fusion categories and \mathcal{C}_{int} cannot be modular, so $\mathcal{Z}_2(\mathcal{C}_{int}) = \langle g \rangle$ for some non-trivial self-dual invertible $g \in \mathcal{C}_{ad}$. $\langle g \rangle$ is symmetric, so it is either Tannakian or sVec.

Case i: $\langle g \rangle = \text{sVec}$

We have $\langle g \rangle = \text{sVec} = \mathcal{C}'_{int}$, so g cannot stabilize any simple objects in \mathcal{C}_{int} . Then by parity arguments, all simple objects of dimension 2 or 4 must each be stabilized by exactly one nontrivial self-dual invertible simple object. Thus all invertibles must be self-dual, and \mathcal{C}_{ad} must contain at least one simple object of dimension 2.

Case ii: $\langle g \rangle = \text{Rep}(\mathbb{Z}_2)$

Let a tilde denote the number of simple objects of each dimension that are stabilized by g . Then $a' = 2 + 2\tilde{b}$, $b' = \frac{b-\tilde{b}}{2} + 2\tilde{c}$, and $c' = \frac{c-\tilde{c}}{2}$.

$\mathcal{C}_{\mathbb{Z}_2}$ is weakly integral, so g cannot stabilize any simple objects of dimension $\sqrt{2}, \sqrt{p}$, or $\sqrt{2p}$. We have five subcases, given by the possible dimensions of the non-integral simple objects.

Case iia: $m = 4$

Then $m' = 2$.

Case iib: $h = 8$

Then $h' = 4$.

Case iic: $k = 2$

Then $k' = 1$ or $h' = 4$.

Case iid: $h = 4$ and $k = 1$

All invertibles must stabilize the only simple object of dimension $2\sqrt{p}$, so $h' = 4$.

Case iie: $4|f$ and $d = \frac{4p-f}{4}$

Then $f' = \frac{f}{2} + 2\tilde{d}$ and $d' = \frac{d-\tilde{d}}{2}$.

4 Future Work

It remains to fully classify the cases where $a = 4$. Once $\text{FPDim}(\mathcal{C}) = 16p$ has been fully classified, examining the related case of $\text{FPDim}(\mathcal{C}) = 32p$ could illuminate generalizations to $2^n p$.

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