

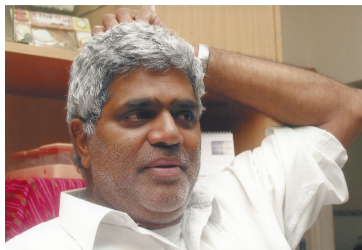
# Mathematical Interests of Kiran Chilakamarri

Ken W. Smith  
Sam Houston State University

CombinaTexas, May 2016

# Kiran Chilakamarri

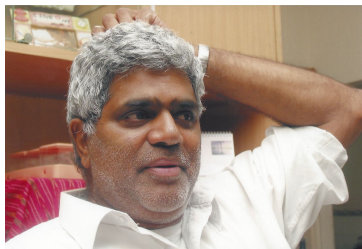
Kiran Babu Chilakamarri passed away on April 25, 2015, at the age of 62. He was a professor at Texas Southern University and a member of the MAA since 2014. He specialized in graph theory, although his research applications spanned many mathematics and scientific domains. He earned two PhDs and authored over 30 papers, many in collaboration.



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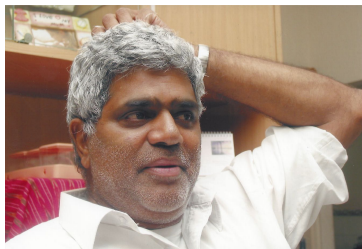


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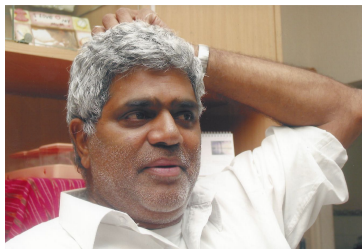
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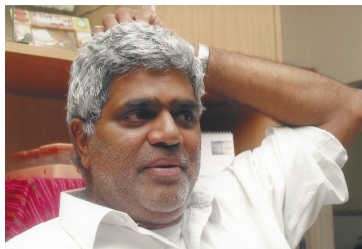
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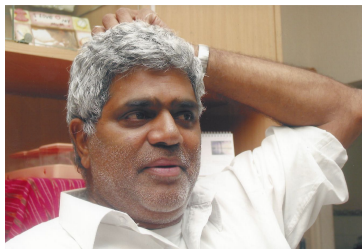
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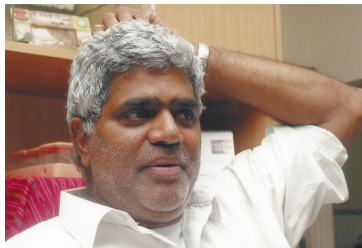
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# Kiran Chilakamarri









# Unit Distance Graphs

Kiran's interest were varied. He published 10 or more papers on unit distance graphs.

Consider the plane  $\mathbb{R}^2$  with ordered pairs adjacent if and only if their distance is 1.

Call this graph  $(\mathbb{R}^2, 1)$ .

It has cardinality equal to the continuum; indeed, the degree of any vertex is the continuum.

What is the chromatic number of that graph?

This problem arises in consideration of distance-preserving functions of  $\mathbb{R}^2$ .

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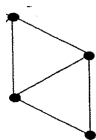
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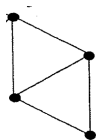
Consider the “diamond”:



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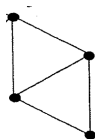
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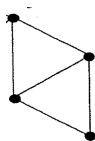
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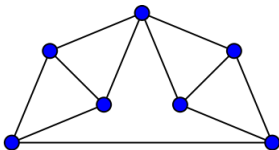
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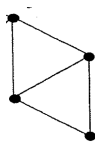
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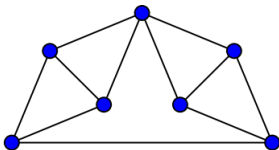
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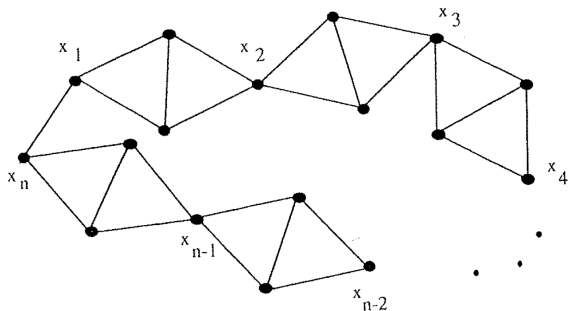
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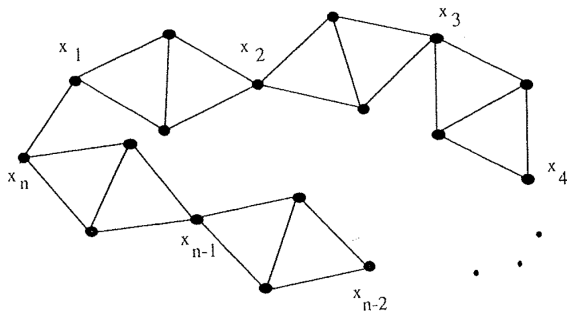
We can 3-color this graph in essentially one way: color the vertices of degree 3 with two colors, say 1, 2 and color the other vertices with color 3.

At the end of the chain, add an edge to the last vertex and the first, creating an edge between the only two vertices of degree 2.

This requires four colors and it is easy to see that this is a unit distance graph. Indeed, with enough diamonds, it is a *matchstick graph*.

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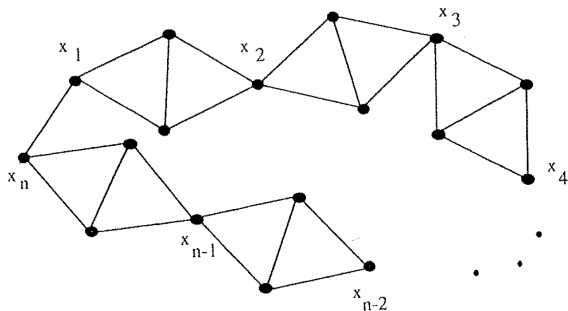
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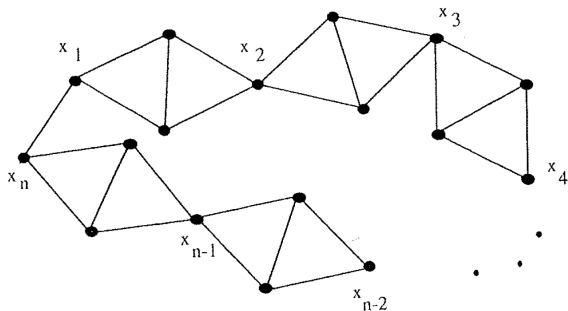
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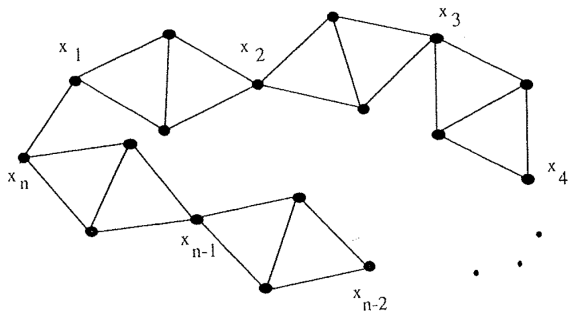


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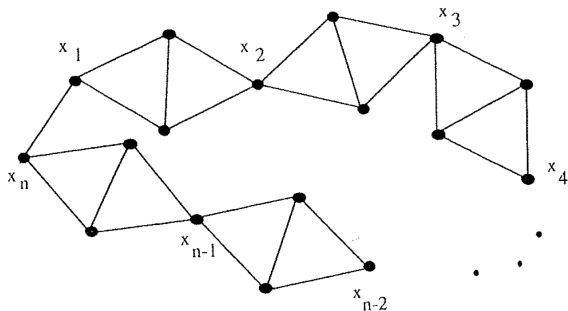
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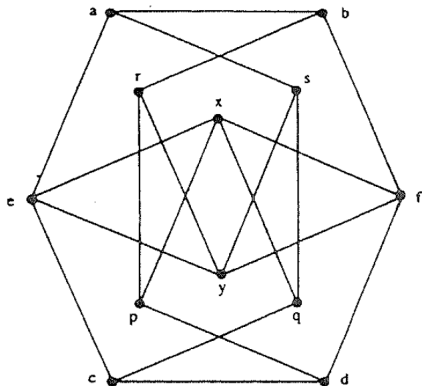
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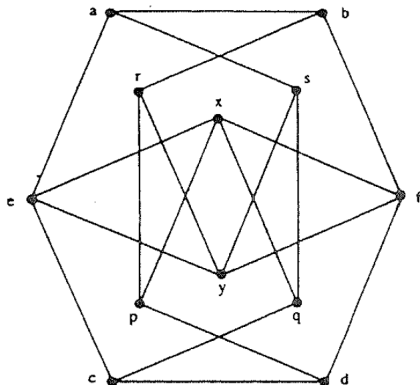
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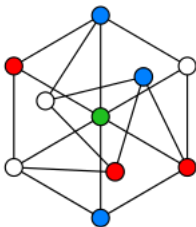
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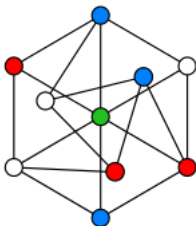
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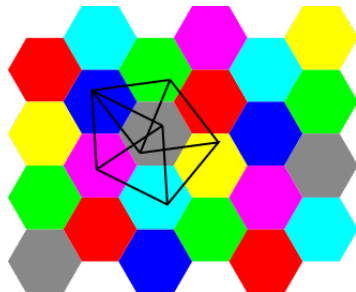


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Tile the plane with hexagons of diameter a little less than 1

7-color the interiors of the hexagons so that no points of distance 1 lie in hexagons of the same color.



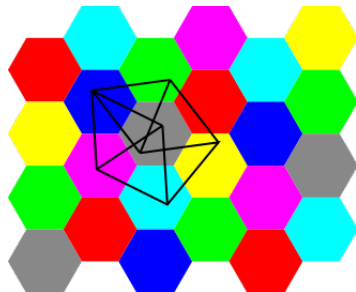
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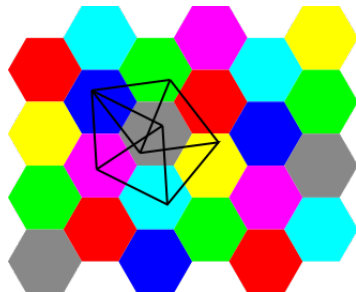


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This leads to some related questions.

Suppose we pick a finite subset  $V$  of  $\mathbb{R}^2$  and look at the induced subgraph under this relation.

Which **finite** graphs are realizable in that manner?

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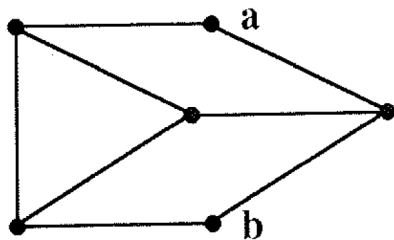
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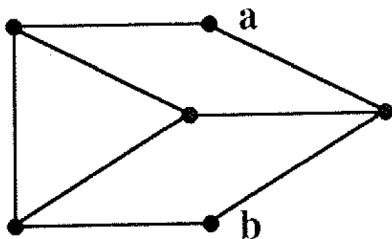
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# Unit Distance Graphs

There are some natural generalizations of this problem.

Replace  $\mathbb{R}^2$  with a metric space of some type, such as  $\mathbb{R}^n$ ,  $\mathbb{Q}^n$  or  $\mathbb{Z}^n$ .

We don't want to generalize too far, as every graph provides a natural metric space for which the graph *is* the unit graph.

But interesting infinite metric spaces provide a challenge. They need not be Euclidean....

Kiran wrote a paper on Minkowski metric spaces and their unit-distance graphs.

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We could also replace unit distance by  $1 \pm \epsilon$ ; think of bonds in atoms, and look at finite graphs.

Given a metric space  $M$ , and distance  $r$ , we can look at the graph  $(M, r)$ .

More generally, let  $(M, r, \epsilon)$  or  $(M, [r - \epsilon, r + \epsilon])$  be the graph with vertices from  $M$ , any pair of vertices are adjacent if and only if their distance is in the closed interval  $[r - \epsilon, r + \epsilon]$ .

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Given a metric space  $M$ , and distance  $r$ , we can look at the graph  $(M, r)$ .

More generally, let  $(M, r, \epsilon)$  or  $(M, [r - \epsilon, r + \epsilon])$  be the graph with vertices from  $M$ , any pair of vertices are adjacent if and only if their distance is in the closed interval  $[r - \epsilon, r + \epsilon]$ .

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It was shown that  $\chi(\mathbb{Q}^4, 1) = 4$  and  $\chi(\mathbb{Q}^5, 1) \geq 5$ .

The value of  $\chi(\mathbb{Q}^n, 1)$  is closely related to  $\chi(\mathbb{Z}^n, r)$  for large values of  $r$ .  
(This result from Kiran.)

Kiran also showed that  $\chi(\mathbb{Q}^5, 1) \geq 6$  and conjectured that  $\chi(\mathbb{Q}^5, 1) = 8$ .

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Erdős, Harary and Tutte define the dimension of a graph to be the smallest  $n$  for which the graph is a unit-distance graph in  $\mathbb{R}^n$ .

They prove that the dimension is less than twice the chromatic number and that  $\chi(\mathbb{R}^n, 1)$  is always finite.

How does  $\chi(\mathbb{R}^n)$  grow? We don't even know its value for  $n = 2$ !

Larman and Rogers:  $\chi(\mathbb{R}^n, 1) \leq (3 + o(1))^n$ , so  $\chi(\mathbb{R}^n, 1)$  is eventually bounded by  $4^n$ .

Kiran found infinite bipartite subgraphs of the unit distance graph in the plane including one with every neighborhood uncountable. (Relied on Zorn's Lemma.)

If  $\mathbb{R}^2$  has a 4-coloring then every open disk of  $\mathbb{R}^2$  uses at least 3 colors.

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Suppose we have a coloring of the plane.

One can fix a color and ask about the set of vertices of that color.

It is possible that such a set could be very strange. It might not be measurable.

Its existence might rely on the axioms of set theory such as Zorn's Lemma or the Axiom of Choice (Maybe even the Continuum Hypothesis??)



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(Elaborate here!)

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He had an article in the Monthly that disproved a conjecture about decompositions of bipartite graphs.

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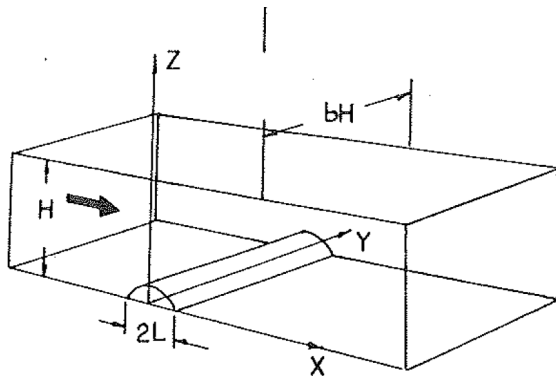
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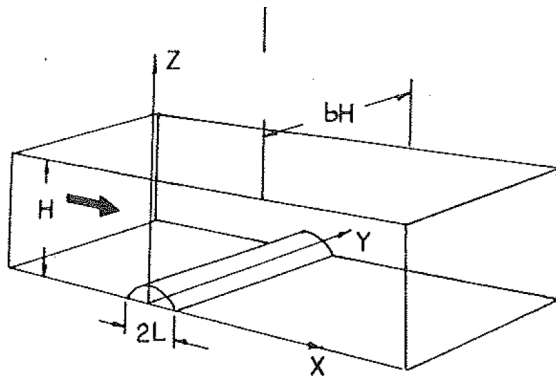
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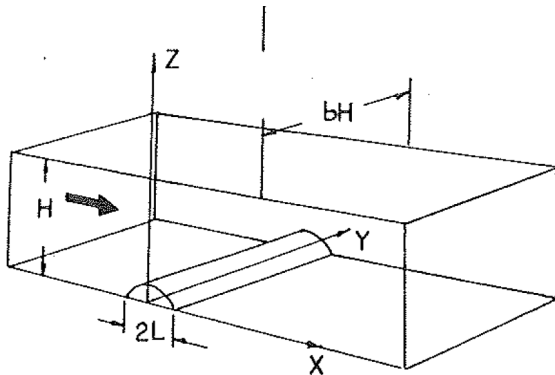




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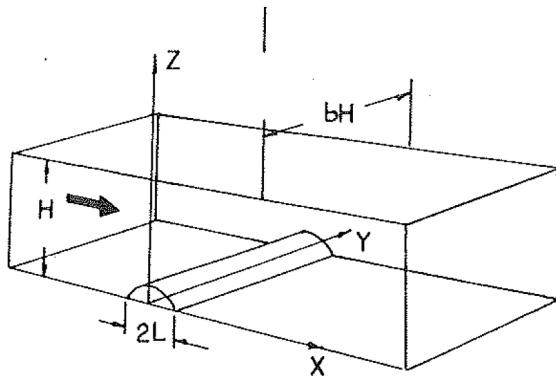
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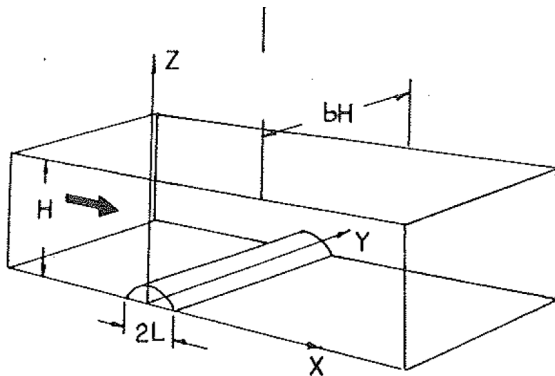
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*Advances in Graph and Matroid Theory*

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