

Open problems raised during the Workshop in Analysis and Probability

July-August 2009

The problems here were either submitted specifically for the purpose of inclusion in this list, or were taken from talks given during the Workshop in Linear Analysis and Probability.

Problem 1 (Submitted by Francisco Garcia). Let X be a normed space.

- (a) Suppose X is strictly convex. Can one renorm it so that its completion is strictly convex?
- (b) Suppose X is locally uniformly convex. Can one renorm it so that its completion is locally uniformly convex?

Problem 2 (Submitted by Mrinal Raghupathi). Let B be a finite Blaschke product, $z_1, \dots, z_n \in \mathbb{D}$ and I the ideal $\{f \in H^\infty : f(z_1) = \dots = f(z_n) = 0\}$. What is the least dimension of a Hilbert space on which the quotient $(\mathbb{C} + B \cdot H^\infty)/I$ can be represented isometrically? In particular, are there finite dimensional ones?

Problem 3 (Submitted by Yun-Su Kim). Does the Riesz representation theorem hold for Hilbert spaces with respect to $L(C(K))$ -valued norms?

Problem 4 (Submitted by Yun-Su Kim). Is every Banach space a Hilbert space with respect to some $L(C(K))$ -valued norm?

Problem 5 (Submitted by Greg Kuperberg). The Mahler conjecture asserts that the product volume $(\text{Vol } K)(\text{Vol } K^\circ)$ is minimized for Hanner polytopes. Is the volume of the starlike body K^\diamond defined in my paper maximized for Hanner polytopes?

Problem 6 (Submitted by Greg Kuperberg). (a) Is the variance $V_{K \times K^\circ}[x \cdot y]$ maximized by ellipsoids?

- (b) In particular, is this conjecture easier for log-concave or s -concave measures in one dimension?

Problem 7 (Submitted by Julio Bernues). Let E be a k -dimensional subspace of \mathbb{R}^n and P_E the orthogonal projection onto E . We want to estimate the isotropy constant of $P_E(B_p^n)$ a projection of the unit ball of ℓ_p^n for $1 < p \leq 2$. In order to do it we consider $\int_{P_E(B_p^n)} f(x) dx$ for good f . An expression for any E and $p = 1$ or E hyperplane and any $1 < p \leq 2$ is known. The question is to extend it to any E and any $1 < p \leq 2$.

For further submissions or corrections, send an email to jcdom@math.tamu.edu

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