Open Problems - Analysis and Probability 2010

Typed by Sofia Ortega Castillo

Summer 2010

Problem 1, by Roland Speicher. Can we distinguish the distribution of elements $I(f)$ and $I(g)$ from different chaos; $f \in L^2(\mathbb{R}_+^n)$, $g \in L^2(\mathbb{R}_+^m)$, with $n \neq m$. What can we say about regularity properties of the distribution of $I(f)$? Can we have atoms? Do we have a density?

Problem Set 2, by Quanlei Fang. The Drury-Arveson space of functions of d variables (denoted as H_d^2) is the reproducing kernel Hilbert space with the normalized reproducing kernel over the unit ball of $\mathbb{C}^d$ given by the formula

$$k_z(w) = \frac{(1-|z|^2)^{1/2}}{1-\langle w, z\rangle}, \qquad |z| < 1, |w| < 1.$$

Suppose $f \in H_d^2$.

- Does the condition
$$\sup_{|z|<1} \|fk_z\| < \infty$$
implies that f is a multiplier for H_d^2?
(Recall that a function $f \in H_d^2$ is said to be a *multiplier* if $fg \in H_d^2$ for every $g \in H_d^2$.).
- Under what condition on f is the commutator $\{M_f^\circ, M_f\}$ compact?
- Under what condition on f does the commutator $\{M_f^\circ, M_f\}$ belong to the Schatten class $\mathcal{C}_p$, $p > n$?
- What is the essential commutant of $\mathcal{TM}_n$?

Problem Set 3, by Detelin Dosev.

- What are the commutators on $(\oplus l_2^n)_{c_0}$ and $(\oplus l_2^n)_{l_1}$?
- Are the compact operators (on any Banach space) always commutators?
- In which spaces is every compact operator a commutator of two compacts? Is it true in l_2?

Problem Set 4, by Piotr W. Nowak.

- What is the distortion of Schreier graphs of the Basilica group?

- What is the asymptotic behavior of $\lambda_1^{(2)}(\Gamma_n)$ for the Schreier graphs associated to the Hanoi Tower groups $\mathbf{H}^{(k)}$ for $k \geq 4$?
- What is the distortion of the family of Schreier graphs $\{\mathbf{H}_n^{(k)}\}_{n\in\mathbf{N}}$ when $k \geq 4$?

Problem Set 5, by Kevin Beanland. In the following, $L(X, Y)$ is equipped with the strong operator topology and X and Y are separable.

- Suppose X is indecomposable. Is $SS(X, Y)$ a Borel subset of $L(X, Y)$?
- Suppose X does not contain an uncoditional basic sequence. Is $SS(X, Y)$ a Borel subset of $L(X, Y)$?
- Suppose ϱ is unbounded on $SS(X, Y)$. Is X a universal space?

Problem Set 6, by Michael Doré.

- What are the possible Hausdorff dimensions of the closed universal differentiability sets?
- If $\dim(X) = \infty$, can we find any similarly small subsets of X that are universal?
- What similar results are there for Lipschitz $f : X \to \mathbb{R}^m$?

Problem Set 7, by Asger Törnquist.

- Is isomorphism of nuclear simple C^* algebras bi-reducible with some other naturally describable equivalence relation, e.g. the $\leq_B$ maximal equivalence relation coming from the actions of some particular Polish group?
- Is isomorphism of nuclear simple C^* algebras Borel reducible to the orbit equivalence relation of a continuous Polish group action.
- Is the isomorphism relation for AF algebras complete for countable structures?
- Is the isomorphism relation for countable torsion free abelian group (of infinite rank!) complete for countable structures?

Problem Set 8, by Ilijas Farah.

- Does $\bigotimes_{\aleph_1} M_2(\mathbb{C})$ embed into $\bigotimes_{\aleph_0} M_2(\mathbb{C}) \otimes \bigotimes_{\aleph_1} M_3(\mathbb{C})$?
(Here $\aleph_1$ stands for the least uncountable cardinal. Note that the question has a negative answer for any smaller pair of cardinals.)
- Does $\bigotimes_{\kappa} M_2(\mathbb{C})$ embed into $\bigotimes_{\aleph_0} M_2(\mathbb{C}) \otimes \bigotimes_{\kappa} M_3(\mathbb{C})$ for all, or any, or some, $\kappa \geq \aleph_1$.?
- Assume A is a simple separable unital C*-algebra and α is its automorphism. Are there necessarily a pure state ϕ and an inner automorphism β of A such that $\phi \circ \alpha = \phi \circ \beta$?

- Given a small enough $\epsilon > 0$, is there $\delta > 0$ such that the following holds?:

  Given a positive integer n and an n-dimensional Hilbert space with orthonormal bases e_i, $i \leq n$ and f_i, $i \leq n$ satisfying $|(e_i|f_j)| < \delta$ for all i and all j there is a projection P spanned by a subset of $\{e_i : i \leq n\}$ such that for every $j \leq n$ we have $\epsilon \leq \|Pf_j\| \leq 1 - \epsilon$.

Problem Set 9, by Todor Tsankov.

- What type of invariant structures does the free group admit?
- Is it true that for every relational, ultrahomogeneous structure with SAP X, there is a copy of $\mathbb{F}_2$ in $Aut(X)$ which acts freely on X?
- Does there exist an oligomorphic group which does not have property (T)? A Roelcke precompact? Polish group?

Problem set 10, by Christian Rosendal.

- Are Borel sets adversarially Ramsey?
- Does the difference of proof theoretical strength between Σ_3^0 and Δ_4^0 determinacy translate into a difference in truth value with respect to our games?

Problem Set 11, by Bunyamin Sari.

- How do we distinguish uniformly non-homeomorphic spaces which have the same local structure? That is, are there reasonably general infinite dimensional invariants?
- Suppose X and Y are uniformly homeomorphic reflexive and separable spaces. Is it true that they have the same asymptotic structure?