

# 2017 Texas Differential Equations Conference Texas A&M Univeristy, College Station, TX

## BOOK OF ABSTRACTS

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**Speaker:** Youn-Sha Chan, University of Houston-Downtown  
**e-mail:** ChanY@uhd.edu  
**Title:** *Dynamical Models on Embryonic Stem-cell Switch*  
**Abstract:** A dynamical modeling on embryonic stem-cells (ESCs) is investigated. As experimental data has lead to an evidence that a genetic regulatory network is responsible for ESC fate decision in both human and mouse, the modeling is focus on how ESCs are regulated at the genetic level. We present several dynamic models to describe the mutual regulation of the genes involved in these networks. Some of the dynamic models we investigated can be used to describe the functionality of a bistable switch, which accounts for the differentiation and self-renew status.

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**Speaker:** Frank Cisneros, TAMU - Corpus Christi  
**e-mail:** fcisneros1@islander.tamucc.edu  
**Title:** *Analytic Calculation of Harmonic Potentials involving Spherically Coated Multiphase Domains*  
**Abstract:** The problem of a conducting sphere with a concentric (dielectric) coating under the influence of an external primary (electrostatic) field leads to multi-phase models in mathematical physics. Such models rely on the calculation of scalar potentials of mixed boundary value problems (BVP) for linear partial differential equations (PDE) in the respective phases. In this talk, we show the formulation of mixed BVPs for potential functions in the context of electrostatics. The vector field equations reduce to scalar Laplace equations for the potentials, say  $\Phi(r, \theta, \phi) - r, \theta, \phi$  being spherical polar coordinates, in the respective phases. The determination of harmonic potentials involve Dirichlet, and Neumann type mixed boundary conditions on the outer spherical surface  $r = b$  and Dirichlet condition at the core  $r = a$ . By the use of variable separable method we attempt to find exact solutions for the mixed boundary value problems involving Legendre functions. Analytic results for three primary fields viz., (i) constant field; (ii) linear field; and (iii) field induced by a point charge will be discussed. Graphical illustrations of the contour plots and force calculations with various physical parameters will also be shown. This is a joint work with D. Palaniappan.

3.

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**Speaker:** Ze Cheng, University of Texas at San Antonio

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**Title:** *The existence of solution to the supercritical Hardy-Littlewood-Sobolev type system*

**Abstract:** This result answers the basic question in the study of HLS system, i.e., all supercritical systems admit solution. Before our result, only some special cases are known to admit solution. The new idea of the proof is to apply recent development in the regularity theory of fractional Laplacian. Comparing to traditional method of studying fractional Laplacian equation, which extends the system to higher dimension half space and transforms to classic Laplacian, our method is more direct and simpler.

4.

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**Speaker:** Sourav Dutta, TAMU

**e-mail:** sdutta@math.tamu.edu

**Title:** *A modern hybrid method for multiphase, multicomponent flow and transport in porous media*

**Abstract:** We will present a fast and efficient approach for solving a coupled system of elliptic and parabolic equations arising in the context of multi-phase flows in porous media. Such flows are found in many different physical applications which include enhanced oil recovery processes, subsurface flows and bio-fluid flows. A new global pressure function for incompressible, multicomponent, immiscible two-phase flows will be introduced. The system of equations using the global pressure model is not as strongly coupled as the models which use the phase pressures as simulation variables. This system is numerically solved using a modern, hybrid method based on a combination of a discontinuous non-traditional finite element formulation and a time-implicit finite difference scheme based on the modified method of characteristics. Results of a theoretical convergence study and numerical comparisons with an exact solution and also with existing literature will be presented. We will conclude with a discussion of the effect of various chemical components and of the heterogeneity of the domain properties on the spontaneous formation of finger patterns and other complex flow characteristics.

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**Speaker:** Mostafa Fazly, University of Texas at San Antonio

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**Title:** *Monotonicity Formulae for Elliptic PDEs* (Canceled)

**Abstract:** I will speak about the importance of monotonicity formulae in the context of elliptic PDEs from various perspectives. Monotonicity formulae are applied to classify solutions of PDEs and they are in close connections with a priori estimates. We present monotonicity formulae for certain local and nonlocal equations. This is partially a joint project with Juncheng Wei (UBC).

6.

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**Speaker:** Zhaosheng Feng, University of Texas - Rio Grande Valley

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**Title:** *Approximate Solutions to the Korteweg-de Vries-Burgers Equation*

**Abstract:** In this talk, we provide a connection between the Abel equation of the first kind, an ordinary differential equation that is cubic in the unknown function, and the Korteweg-de Vries-Burgers equation, a partial differential equation that describes the propagation of waves on liquid-filled elastic tubes. We present an integral form of the Abel equation with the initial condition. By virtue of the integral form and the Banach Contraction Mapping Principle we derive the asymptotic expansion of bounded solutions in the Banach space, and use the asymptotic formula to construct approximate solutions to the Korteweg-de Vries-Burgers equation.

7.

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**Speaker:** Eleftherios Gkioulekas, University of Texas - Rio Grande Valley

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**Title:** *Revisiting the dissipation scales of the energy cascade of 3D turbulence as anomalous scaling functions*

**Abstract:** The usual concept of an energy cascade that has a unique associated dissipation scale is an oversimplification. Aside from the fact that self-similar scaling for higher-order structure functions terminates at different dissipation scales, back in 1996, L'vov and Procaccia noted an additional anomaly; starting from an  $n^{\text{th}}$ -order generalized structure function, consisting of a product of velocity differences, each between two different points, when all velocity difference separations have length scale  $R$  and one velocity difference separation is reduced to a smaller scale  $r$ , the crossover to dissipation range will occur at the scale  $\ell_n(R)$  which is  $R$ -dependent. The fixed point  $\lambda_n$  such that  $\ell_n(\lambda_n) = \lambda_n$  gives the standard dissipation scale associated with the  $n^{\text{th}}$ -order standard structure functions. In my talk, I will make note of an additional anomaly. If, instead of reducing one velocity difference separation, we reduce  $p$  velocity difference separations to scale  $r$ , that defines a different dissipation scale function  $\ell_{np}(R)$  and a different fixed-point  $\lambda_{np}$ . The new anomaly is that  $\lambda_{np}$  is not independent of  $p$ .

8. 

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**Speaker:** Yuliya Gorb, University of Houston

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**Title:** *A Robust Preconditioner for High-Contrast Problems*

**Abstract:** This talk concerns a robust numerical treatment of an elliptic PDE with high contrast coefficients. A finite-element discretization of such an equation yields a linear system whose conditioning worsens as the variations in the values of PDE coefficients become large. We introduce a procedure by which the discrete system obtained from a linear finite element discretization of the given continuum problem is converted into an equivalent linear system of the saddle point type. Then a robust preconditioner for the Lancsoz method of minimized iterations for solving the derived saddle point problem is proposed. Numerical examples demonstrate effectiveness and robustness of the proposed class of preconditioners and show the number of iterations independent of the contrast and the discretization size. This is a joint work with Daria Kurzanova and Yuri Kuznetsov (UH).

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**Speaker:** Cong Gu, Texas A&M University

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**Title:** *Iterative Scaling Method for Numerical Solution of Kirchhoff-type Problems with Cubic Nonlinearity*

**Abstract:** Kirchhoff-type problems involve the  $H^1$ -norm as a perturbation to the coefficient in nonlinear elliptic PDEs. When the perturbation is small, existence of the solution can be proved. The speaker will discuss an iterative scaling algorithm for finding the numerical solutions of the Kirchhoff-type problem with cubic nonlinearity. As an added benefit, the maximum level of the perturbation allowed can be quantified in the process.

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**Speaker:** Changfeng Gui, University of Texas at San Antonio

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**Title:** *The Sphere Covering Inequality and its application to a Moser-Trudinger type inequality and mean field equations*

**Abstract:** In this talk, I will introduce a new geometric inequality: the Sphere Covering Inequality. The inequality states that the total area of two *distinct* surfaces with Gaussian curvature less than 1, which are also conformal to the Euclidean unit disk with the same conformal factor on the boundary, must be at least  $4\pi$ . In other words, the areas of these surfaces must cover the whole unit sphere after a proper rearrangement. We apply the Sphere Covering Inequality to show the best constant of a Moser-Trudinger type inequality conjectured by A. Chang and P. Yang. Other applications of this inequality include the classification of certain Onsager vortices on the sphere, the radially symmetry of solutions to Gaussian curvature equation on the plane, classification of solutions for mean field equations on flat tori and the standard sphere, etc. The resolution of several open problems in these areas will be presented. The work is jointly done with Amir Moradifam from UC Riverside.

11.

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**Speaker:** Qi Han, Texas A&M - San Antonio

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**Title:** *On the first exterior  $p$ -harmonic Steklov eigenvalue*

**Abstract:** Let  $U \subsetneq \mathbb{R}^N$  be an exterior region with  $0 \notin U$  when  $N \geq 3$ , and  $\partial U$  be the union of finitely many disjoint, closed, Lipschitz surfaces with the total surface area 1. Let  $E^{1,p}(U)$  be the space of functions with  $u \in L^{p^*}(U)$  and  $|\nabla u| \in L^p(U)$  for  $p := \frac{pN}{N-p}$  when  $1 < p < N$ .  $E^{1,p}(U)$  is continuously embedded into the space  $L^q(\partial U, d\sigma)$  when  $1 \leq q \leq p_* := \frac{p(N-1)}{N-p}$  and when  $1 \leq q < p_*$ , this embedding is also compact. Define the **Rayleighs quotient**  $\delta(q) := \inf_{u \in E^{1,p}(U)} \frac{\|u\|_{\nabla}^p}{\|u\|_{q,\partial U}^p}$ . Then, one has  $\delta(q) : [1, p_*] \rightarrow (0, \infty)$  is an absolutely continuous function and thus  $\delta(q) \in W^{1,1}([1, p_*])$ .

12.

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**Speaker:** Isaac Harris, Texas A&M University

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**Title:** *Direct Method for Reconstructing Inclusions from Electrostatic Data*

**Abstract:** Abstract: In this talk, we will discuss the use of the Linear Sampling Method to reconstruct impenetrable inclusions from Electrostatic Cauchy data. We consider the case of a perfectly conducting and impedance inclusion. In either case we see that the Dirichlet to Neumann mapping can be used to reconstruct impenetrable sub-regions. We also propose a non-iterative method to reconstruct the impedance parameter from the knowledge of multiple Cauchy pairs which can be computed from Dirichlet to Neumann mapping. Some numerical reconstructions will be presented in two dimensions.

13.

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**Speaker:** Vu Hoang, Rice University

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**Title:** *Blowup for model equations of fluid mechanics*

**Abstract:** In this talk, I consider the 2D inviscid Boussinesq equations in vorticity form. It remains a challenge to decide if finite time blowup happens for smooth initial data or not. I introduce a model problem for the Boussinesq equations associated to the hyperbolic flow scenario for which we can show finite-time blowup (joint work with B. Orcan, M. Radosz, H. Yang).

14.

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**Speaker:** Yeyao Hu, University of Texas at San Antonio

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**Title:** *Stationary solutions to the Ohta-Kawasaki problem on closed surfaces*

**Abstract:** In this talk, we will discuss an inhibitory system. The system arises from the Ohta-Kawasaki density functional theory concerning the morphology of diblock copolymers. We study the sharp interface limit of the Ohta-Kawasaki energy functional and identify it as a nonlocal geometric variational problem. Stationary disc assemblies are discovered as critical points of the sharp interface energy on planar domains and on two dimensional closed manifolds.

15.

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**Speaker:** Akif Ibragimov, Texas Tech University

**e-mail:** akif.ibragimov@ttu.edu

**Title:** *On positive solution in narrowing at infinity domain for non-divergent equations of second order*

**Abstract:** In my talk I will present new results for solutions of Zaremba type problem with respect to non-divergent equations with discontinue coefficients of second order in unbounded domain. We will prove that if Neuman boundary is in  $C^2$  then positive solution with homogeneous boundary conditions will converge to infinity with rate depending on capacity of Dirichlet Data, and how fast domain is "narrowing" at infinity. No additional constraint imposed on Dirichlet part of the boundary. Obtained results and estimates generalize Mazya's test proven for equations in divergent form. Proof is based on new lemma of growth for the solutions of equation with mixed boundary conditions in cylindrical layers. From obtained results Phragmén-Lindelöf principle follows for degenerate at infinity equations with mixed boundary conditions. Corresponding estimates depend on rate of divergence of the operator and rate of "narrowing" of domain at infinity. This talk is based on a joint research with Cao Dat, Texas Tech University, Akif Ibragimov, Texas Tech University and Alexander Nazarov, Saint-Petersburg University, and Russian Academy of Science.

16.

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**Speaker:** Bingbing Ji, Texas A&M University

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**Title:** *A Local Minimax Method Using Nehari Manifold for Finding Differential Saddles*

**Abstract:** A new local minimax method (LMM) for finding the first few unconstrained saddles of a functional is developed, so that different types of saddle point problems in infinite-dimensional spaces can be solved. This method is based on a dynamics of points on virtual geometric objects such as curves, surfaces, etc. and it covers several existing algorithms in the literature as its the mathematical framework is general. The algorithm is mathematically justified by establishing a strong energy dissipation law and showing the convergence of the algorithm. It is applied to numerically compute saddles of a semilinear elliptic PDE for both (focusing) M-type and (defocusing) W-type cases. It is shown that those virtual geometric objects can be easily defined without knowing their explicit expressions and extended to find k-saddles so there is a great flexibility to choose preferred geometric objects for some purposes, such as convergence acceleration. Inspired by this feature, Nehari manifold is used to accelerate the convergence and a comparison of convergence speed on the same semilinear elliptic PDE with quadratic geometric objects is given, then a mixed M and W type case is solved by LMM with Nehari manifold. This is a joint work with Jianxin Zhou.

17.

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**Speaker:** Janak Joshi, University of North Texas

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**Title:** *Existence and Nonexistence of Solutions for Sublinear Problems with Prescribed Number of Zeros on Exterior Domains* (Canceled)

**Abstract:** We prove existence of radial solutions of  $\Delta u + K(r)f(u) = 0$  on the exterior of the ball of radius  $R$  centered at the origin in  $\mathbb{R}^N$  such that  $\lim_{r \rightarrow \infty} u(r) = 0$  if  $R > 0$  is sufficiently small. We assume  $f : \mathbb{R} \rightarrow \mathbb{R}$  is odd and there exists a  $\beta > 0$  with  $f < 0$  on  $(0, \beta)$ ,  $f > 0$  on  $(\beta, \infty)$  with  $f$  sublinear for large  $u$ , and  $K(r) \sim r^{-\alpha}$  for large  $r$  with  $\alpha > 2(N - 1)$ . We also prove nonexistence if  $R > 0$  is sufficiently large.

18.

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**Speaker:** Steven D. London, University of Houston-Downtown

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**Title:** *Solitary waves in rotating shallow water magnetohydrodynamics*

**Abstract:** We consider a perfect electrically conducting rotating fluid in the presence of an ambient toroidal magnetic field, governed by the shallow water magnetohydrodynamic (MHD) equations in a modified equatorial beta plane approximation. Using a multiple scale asymptotic technique, previously developed by Boyd (1980) for equatorial solitary hydrodynamic waves, we look for solitary MHD waves. In the case of a weak ambient magnetic field, the leading order equations governing the poleward velocities can be solved using associated Legendre functions. These functions are multiplied by an amplitude function of slow length and time variables. They are solved for at second order via a compatibility condition and have the form of equatorial Rossby solitary waves. When the ambient magnetic field is moderately strong, the equations governing the poleward velocities cannot be solved using special functions. Instead, we are able to apply a WKB type approximation to solve this problem using a large parameter which arises naturally in the governing equations. The solution has the form of an equatorial magneto-Rossby solitary wave. When the ambient magnetic field is very strong, the solutions are bounded away from the equator in the form of magnetostrophic solitary waves. The possible relationship between all of these waves and solar phenomena such as the solar cycle, sunspots and active longitudes is discussed.

19.

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**Speaker:** Taoufik Meklachi, Penn State University, Harrisburg.

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**Title:** *Vibration control in spring mass systems*

**Abstract:** In this paper, we are introducing a new method to solve active vibration suppression problems in a 1D network of an arbitrary number of coupled spring- mass units connected in series. The spring-mass system is used in many applications in a wide range of fields, for instance in defense detection/cloaking strategies, biomedical engineering, structures engineering, computer graphics, acoustics, etc... The speed and accuracy of the intervening active control is a key factor to the success of the vibration suppression scheme, whether it is an incoming detecting wave, an earthquake vibration or an acoustic noise for instance.

20.

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**Speaker:** D. Palaniappan, TAMU - Corpus Christi

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**Title:** *General Solution Representations for the Motion of a Circular Disk in a Viscous Fluid*

**Abstract:** The slow motion of a circular disk in a viscous fluid in the limit of zero Reynolds number (Stokes flow) is studied recently using a solution representation with *two harmonic functions*. In this talk, we show two general solution representations for vector Stokes equations for solving general motion of a disk. The first solution for the velocity vector field  $u$  is expressed in terms of biharmonic and harmonic scalar functions  $\{A, B\}$  while the second involves three scalar harmonic functions  $\{\Pi, \Psi, \chi\}$ . Our solution forms indicate that in general, three independent harmonic functions are necessary in order to study an arbitrary motion of a disk in creeping flow conditions. Our representation reveals connections to some well-known solutions due to Galerkin and Papkovitch-Neuber. We discuss exact solutions of a few boundary-value problems based on our general solution. The boundary conditions lead to dual integral equations involving Bessel functions whose solutions can be found in certain cases. Analytic results for the translation and rotation of a disk in a viscous fluid and expressions for the force/torque will be shown as well.

21. 

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**Speaker:** Du Pham, University of Texas at San Antonio

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**Title:** *On weak solutions of the Shigesada-Kawasaki-Teramoto equations*

**Abstract:** Cross-diffusion systems have rich applications in biology, ecology, and physics. Due to the lacking of the maximum/comparison principle, current methods of finding weak solutions for these cross-diffusion systems usually assume a change of variables, the entropy method. In this talk, I will discuss the existence of weak solutions of Shigesada-Kawasaki-Teramoto (SKT) equations using finite difference approximations in time, cut-off functions and compactness arguments. The weak attractor is established in the settings of the weak solutions of the Navier-Stokes equations in three-dimensional space where the uniqueness is not assumed.

22. 

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**Speaker:** Saroj Pradhan, Southern Methodist University

**e-mail:** spradhan@mail.smu.edu

**Title:** *Parameter Identification of Human Respiratory System with Peripheral and Central Control*

**Abstract:** In this paper we study parameter identification issues by computational means for a system of three nonlinear, parameter dependent delay differential equations with two transport delays with peripheral and central control loops representing a model of human respiratory system. We are especially interested in the identification of the transport delays and peripheral and central control gains. These parameters are not measurable directly. Several case studies are included.

23. 

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**Speaker:** Zhijun (George) Qiao, University of Texas - Rio Grande Valley (UTRGV)

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**Title:** *Negative integrable systems and soliton equations*

**Abstract:** In this talk, I will show how to generate a negative order integrable hierarchy from the Lenard recursion operators, and then find the Lax pair for the entire hierarchy to guarantee the integrability. Interesting thing is that the peakon equation would be coming from the negative hierarchy. I will show some amazing examples including the CH, Negative KdV, two-component short pulse equations etc. Particularly, real and complex short pulse equations can be derived from my earlier works done in the late 90's.

24.

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**Speaker:** Erwin Suazo, University of Texas - Rio Grande Valley

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**Title:** *On Ermakov-Pinney Equation and Soliton Solutions for Partial Differential Equations*

**Abstract:** By means of similarity transformations we study exact analytical solutions for a generalized nonlinear Schrödinger equation with variable coefficients. This equation appears in literature describing the evolution of coherent light in a nonlinear Kerr medium, Bose-Einstein condensates phenomena and high intensity pulse propagation in optical fibers. By restricting the coefficients to satisfy Ermakov-Riccati systems with multiparameter solutions, we present conditions for existence of explicit solutions with singularities and a family of oscillating periodic soliton-type solutions.

25.

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**Speaker:** Barbara A. Shipman, University of Texas at Arlington

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**Title:** *Integrable Moving Frames for Spacelike, Timelike, and Euclidean Surfaces*

**Abstract:** Starting with the standard orthonormal basis in Euclidean or Lorentzian 3-space, we create isothermal moving frames for surfaces by applying to the standard constant frame a scaling and an isometry that varies with the coordinates on the surface. With the appropriate integrability conditions and Lie algebraic viewpoint, we obtain representations of Weierstrass-Enneper type for general spacelike, timelike, and Euclidean surfaces. This is work with Patrick Shipman and David Packard.

26.

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**Speaker:** Stephen Shipman, Louisiana State University

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**Title:** *Short-Time Behavior of the Exciton-Polariton Equations*

**Abstract:** In the exciton-polariton (EP) system, a linear dispersive photon field is coupled to a nonlinear exciton field. Short-time analysis of the lossless system shows that, when the photon field is excited, the time required for that field to exhibit nonlinear effects is longer than the time required for the nonlinear Schrödinger (NLS) equation, in which the photon field itself is nonlinear. For fixed initial data, nonlinear effects of order  $\epsilon$  are observed at time  $t = \epsilon^{1/5}$ , as compared to NLS, for which nonlinear effects are observed at time  $\epsilon$ . These power laws are generalized to initial data of order  $\epsilon^\alpha$ . This is joint work with Cristi Guevara.

27. 

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**Speaker:** Changhui Tan, Rice University

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**Title:** *Global regularity for Burgers equation with density dependent fractional dissipation*

**Abstract:** Fractional equations are a family of equations which connect inviscid and viscous Burgers equation. It is well-known that if the dissipation is strong, the solution is globally regular. On the other hand, if the dissipation is weak (called supercritical case), solution can lose regularity in finite time. In this talk, I will introduce a model where the dissipation depends on density. The model is motivated by self-organized dynamics in math biology. Despite that the equation has a lot of similarities to fractional Burgers equation, the solution is globally regular, even in the supercritical case. I will explain the regularization mechanism that is due to the nonlocal nonlinear modulation of dissipation. This is a joint work with T. Do, A. Kiselev and L. Ryzhik.

28. 

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**Speaker:** Vesselin Vatchev, UT-RGV

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**Title:** *Empirical Approximations of Differential Operators*

**Abstract:** Many important physical quantities are defined as parameters or solutions of differential equations. Examples that we consider are instantaneous frequency and instantaneous bandwidth. In the talk we will discuss how these temporal and spatial quantities can be recovered from empirical data by approximating differential operators by local and simpler operators. Application to N-soliton solution to KdV equations will be presented.

**Speaker:** Ben Wongsaijai, Chiang Mai University, Thailand

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**Title:** *Numerical Analysis of a linear four-level finite difference scheme for the symmetric regularized long wave equation*

**Abstract:** A numerical model to obtain the solution of the symmetric regularized long wave equation is presented. The numerical tool is developed by using a four-level average difference technique in time for solving the fluid velocity independently from the density. At the current point, the approximate solution is easily solved by using the proposed method since it does not require an extra effort to deal with a nonlinear term and density. The fundamental conservative properties of the equation are preserved by the presented numerical scheme, and the existence and uniqueness of the numerical solution are proved. Moreover, the convergence and stability of the numerical solution are also shown. The numerical experiments show that the proposed method improves the accuracy of the solution significantly.

**Speaker:** Wenjing Zhu, Zhejiang Normal University, China

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**Title:** *Exact Traveling Wave Solutions and Bifurcations for a Shallow Water Equation Modeling Surface Waves of Moderate Amplitude*

**Abstract:** In this paper, we consider the traveling wave solutions for a shallow water equation. The corresponding traveling wave systems are a singular planar dynamical systems with one singular straight line. On the basis of the theory of the singular traveling wave systems, we obtain the bifurcations of phase portraits and explicit exact parametric representations for solitary wave solutions and smooth periodic wave solutions, as well as periodic peakon solutions. We show the existence of compacton solutions of the equation under different parameter conditions.