1. A city has an **annual** population growth rate of 2%. Assuming exponential growth, how many years will it take the population to double? (to the nearest year)
   
   a) 8 years  
   b) 17 years  
   c) 23 years  
   d) 35 years  
   e) none of the above

2. Given the equation \( P = 181.6 \times (0.95)^t \), convert this to the form \( P = P_0 e^{kt} \).
   
   a) \( P = 5.20 e^{2.59t} \)  
   b) \( P = 172.52 e^{0.051t} \)  
   c) \( P = 181.6 e^{-0.051t} \)  
   d) \( P = 181.6 e^{2.59t} \)  
   e) none of the above

3. What is the effective annual yield of an investment paying at a 10% annual rate, compounded quarterly?
   
   a) 10.38 %  
   b) 11.05 %  
   c) 10.47 %  
   d) 10.52 %  
   e) none of the above

4. Simplify the expression: \( \frac{\ln(x^2 - y^2) - \ln(x + y)}{\ln((x-y)^2)} \)

5. Which formula best represents the relationship shown in the table?

<table>
<thead>
<tr>
<th>x</th>
<th>2</th>
<th>2.2</th>
<th>2.4</th>
<th>2.6</th>
<th>2.8</th>
</tr>
</thead>
<tbody>
<tr>
<td>f(x)</td>
<td>2</td>
<td>2.42</td>
<td>2.88</td>
<td>3.38</td>
<td>3.92</td>
</tr>
</tbody>
</table>

   a) \( f(x) = mx + b \)  
   b) \( f(x) = ax^2 \)  
   c) \( f(x) = ax^3 \)  
   d) \( f(x) = a \cdot b^x \)

6. Use your calculator to find all intersection points of \( y = 2^x \) and \( y = x^3 \). Round to 3 decimal places. Which function dominates as \( x \) approaches infinity?
7. Choose the function which represents the graph given below. The period is \( \frac{\pi}{3} \).

a) \( f(x) = 3 \cos(6x) - 1 \)

b) \( f(x) = -3 \sin(6x) - 1 \)

c) \( f(x) = -3 \cos(6x) - 1 \)

d) \( f(x) = 3 \sin(6x) \)

e) none of the above

8. If \( f(x) = 10^x \) and \( g(x) = \log x \), what is \( g(f(x)) \)? (composition of functions)

a) \( 10^x \cdot \log x \)

b) \( x \)

c) 1

d) \( x \cdot \log x \)

e) none of the above

9. Choose the function which represents the graph given below.

a) \( f(x) = 0.3(x + 3)(x - 2)(x - 5) \)

b) \( f(x) = -0.15(x + 3)(x - 2)^2 (x - 5) \)

c) \( f(x) = -0.3(x - 3)(x + 2)(x + 5) \)

d) \( f(x) = 9(x + 3)(x - 2)^2 (x - 5) \)

e) none of the above

x-intercepts are -3, 2, and 5. y-intercept is 9.
10. The number of bacteria (in millions) in a bottle of milk after \( t \) days is given in the table below:

<table>
<thead>
<tr>
<th>day</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td># of bacteria present</td>
<td>500</td>
<td>561</td>
<td>630</td>
<td>707</td>
<td>793</td>
</tr>
</tbody>
</table>

Determine a formula of the form \( Q = Q_0e^{kt} \) that gives the number of bacteria present, \( Q \) (in millions), as a function of \( t \) days. If the milk cannot be safely consumed when the bacteria count is greater than 3 billion per bottle, how many days will the bottle of milk be safe to drink?

11. Suppose \( f \), \( g \), and \( h \) are functions of \( x \) such that one of them is proportional to \( x \), one is inversely proportional to \( x \), and one is proportional to the square of \( x \). Using the table below, write \( f \), \( g \), and \( h \) as functions of \( x \) and find the constants of proportionality:

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
<th>( g(x) )</th>
<th>( h(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>600</td>
<td>50</td>
<td>1.25</td>
</tr>
<tr>
<td>10</td>
<td>300</td>
<td>200</td>
<td>2.50</td>
</tr>
<tr>
<td>15</td>
<td>200</td>
<td>450</td>
<td>3.75</td>
</tr>
<tr>
<td>20</td>
<td>150</td>
<td>800</td>
<td>5.00</td>
</tr>
<tr>
<td>25</td>
<td>120</td>
<td>1250</td>
<td>6.25</td>
</tr>
</tbody>
</table>

12. The profit function for a skateboard company is given by \( P(x) = -x^2 + 70x - 125 \), where \( x \) is the price charged by the company for a skateboard.

a) Find the price that will maximize profits.

b) For what prices will the company make a profit?

13. What nominal interest rate has an effective annual yield of 5% under continuous compounding? Round the percent to 2 places.

14. An animal skull still has 20% of the carbon-14 that was present when the animal died. The half-life of carbon-14 is 5730 years. Find the approximate age of the skull (to the nearest year).

15. The depth of water in a tank oscillates once every 6 hours around an average depth of 7 feet. If the smallest depth is 5.5 feet and the largest depth is 8.5 feet, find a formula for the depth in terms of time, measured in hours. Assume the water level starts at 5.5 feet.
16. A female boll weevil can lay on average 200 eggs (source: North Carolina Department of Agriculture). Suppose, for simplicity’s sake, that each boll weevil results in 75 boll weevils the following year. Let $y$ be the number of boll weevils in a particular field in 1995.

a) Suppose there were only 2 boll weevils in a particular field in 1995. Give an exponential model for $y$.

b) According to the model, how many boll weevils would be in that field in the year 2000?

17. Your grandparents put aside $10,000 in an account, which earns 8% interest for your college, when you were born.

a) How much would you have in the account in 18 years if interest was compounded monthly?

b) When would you have $40,000 in the account if the interest was compounded continuously?

18. Describe how the graph of $y = 2f(x - 3) + 4$ differs from the graph of $y = f(x)$.

19. If $g(x) = x^2 - x$, find and simplify $g(x + h) - g(x)$.

20. Determine functions $f$ and $g$ such that $f(g(x)) = e^{3x-2}$. (Do not choose $f(x) = x$ or $g(x) = x$.)

21. The model $y = 5 + 4.9 \cos(\frac{\pi}{6} t)$ represents the water level $y$ in feet at Padre Island $t$ hours after midnight.

a) Find the range of the water level at Padre Island.

b) Find the period of this function to find the amount of time between high tides.

c) How high is the water level at 1:30pm according to the model?

22. The population in country A is 40 million on January 1, 1990 and increases by 6% each year thereafter. The population in country B is 60 million on January 1, 1990 and increases by 4% each year thereafter. How long will it take for the population in country A to surpass the population in country B?

23. A local bank is offering a savings account with an annual interest rate of 5.69% compounded quarterly. What should be the annual interest rate of an account whose interest is compounded continuously, but which has the same effective annual yield?

24. If the population of the world was 2.564 billion in 1950 and the annual population growth rate was 1.85%, find an exponential equation to model the population of the world as a function of time (since 1950). According to your model, during what year will the population first reach 7 billion?
25. Given the following graphs, find their equations. Use the form \( y = f(x) \). Round to 3 decimal places.

a) Points given: \((0, 2)\) and \((2, 1)\)

b) Points given: \((0, 8)\) and \((3, 0)\)

c) Points given: \((-3, 0), (1, 0), (5, 0)\) and \((0, -2.5)\)

d) The period is \( \frac{\pi}{4} \) and the \( y \)-intercept is \((0, 3)\).
1. D
2. C
3. A
4. 0.5
5. B
6. (1.373, 2.591) and (9.940, 981.970); 2^x
7. C
8. B
9. B
10. $Q = 500e^{0.315t}$, about 15 days.
11. $f(x) = \frac{3000}{x}$, $g(x) = 2x^2$, $h(x) = 0.25x$
12. a) $35$  
    b) $1.83 < x < 68.17$
13. 4.88%
14. 13,305 years
15. $y = -1.5\cos(\frac{t}{5}) + 7$ where $y$ is depth in feet and $t$ is time in hours.
16. a) $y = 2(75)^t$  
    b) 4,746,093,750
17. a) $\$42,005.74$  
    b) 17.3 years
18. stretched by a factor of 2 (longer), shifted 3 units right and 4 units up.
19. $2xh + h^2 - h$
20. $f(x) = e^x$ and $g(x) = 3x - 2$
21. a) $0.1 \leq y \leq 9.9$  
    b) 12 hours  
    c) 8.46 feet
22. 21.29 years
23. 5.65%
24. 2004
25. a) $y = 2(0.707)^x$
    b) $y = \frac{1}{3}(x - 3)^2$
    c) $y = \frac{1}{6}(x + 3)(x - 1)^2(x - 5)$
    d) $y = -3\sin(8x) + 3$