Given the graph of \( f'(x) \), answer the following 2 questions:

**NOTE:** A is a root at (-5,0), B is a min at (-3.5,-2), C is a root at (-2,0), D is (-1, 2), E is a max at (2, 6), and F is a root at (4,0). (These should be labeled on the graph of the derivative below)

1. Which of the labeled point(s) on the graph have \( x \)-values that are inflection points for \( f(x) \)?
   a. D
   b. B and E
   c. A, C, and F
   d. C and E
   e. none of the above

2. Which of the labeled point(s) on the graph have \( x \)-values that are local maxima for \( f(x) \)?
   a. C
   b. E
   c. A, C, and F
   d. A and F
   e. none of the above
3. In a lab experiment, some mice are injected with a drug. The concentration of the drug in the bloodstream of the mice can be modeled by \( C(t) = -2t^3 + 6t^2 + 5t \), where \( t \) is in hours since the drug was administered. Find the bioavailability of the drug 3 hours after injection.

   a. 15
   b. 36
   c. 13
   d. 117
   e. none of the above

4. If \( f''(x) = 2x^2 - 1 \) and \( f \) has critical values \( x = -2, x = 0, \) and \( x = 1 \), use the Second Derivative Test to determine which critical value(s) give a local minimum of \( f(x) \).

   a. \( x = 1 \)
   b. \( x = -2, x = 0 \)
   c. \( x = -2 \)
   d. \( x = 0, x = 1 \)
   e. none of the above

5. Assume that during the first three minutes after a foreign substance is introduced into the blood, the rate \( R(t) \) at which new antibodies are produced (in thousands of antibodies per minute) is given by \( R(t) = \frac{t}{t^2 + 1} \), where \( t \) is in minutes. Find the total quantity of new antibodies in the blood at the end of three minutes.

   a. 115 antibodies
   b. 300 antibodies
   c. 1151 antibodies
   d. 1501 antibodies
   e. none of the above
6. Find the first derivative of $f(x) = 2xe^x$.
   a. $2e^x$
   b. $2(e^x + 1)$
   c. $2e^x + 2x$
   d. $2xe^x + 2e^x$
   e. none of the above

7. Given below is the graph of $f'(x)$. It has roots at $x = 2$ and $x = 4$. The values of the two shaded areas under the curve are 9 (larger portion above $x$-axis) and 3 (part below $x$-axis). If $f''(0) < 0$, find $f''(2)$ and $f''(4)$ and tell whether each is a local maximum, local minimum (or neither) of $f(x)$.

8. Find the derivative of the following functions. You do NOT need to simplify your answers.
   a. $f(x) = (\sqrt[3]{x} + 2x)^3$
   b. $f(x) = -2 \cos x + 3 \sin(4x^2 + 1)$
   c. $f(x) = e^{-2x^2} + \ln(-9x^3 + 3x + 1)$
   d. $f(x) = x^6 + e^{-x} + 6 \cdot 4^x$
   e. $f(x) = \frac{1}{x} - \pi^2 \ln(5x^2) + \pi x$
9. Given \( f(x) = x^4 - 4x^3 + 10 \), answer parts a) - g).

   a. Find the critical values of \( f \).
   b. Find the intervals where \( f \) is increasing and decreasing, and give the \( x \)-values of the relative (local) maxima and minima, if they exist.
   c. Give the intervals where \( f \) is concave up and concave down, and give the \( x \)-values of the inflection point(s), if they exist.
   d. Sketch a graph of \( f(x) \). Label the \( x \) and \( y \) values of each relative max, min, and inflection point.
   e. Does \( f \) have a global max on \((-\infty, \infty)\)? If so, where?
   f. Does \( f \) have a global min on \((-\infty, \infty)\)? If so, where?
   g. Does \( f \) have a global min on \([0, 2]\)? If so, where?

10. A company’s revenue from car sales, \( S \) (in thousands of dollars) is a function of their advertising expenditure, \( a \) (in thousands of dollars). Suppose \( S(a) = -5a^2 + 300a \).

   a. What is the revenue from sales if \$6000 is spent on advertising? (include units with your answer)
   b. Find \( S'(40) \). Interpret your answer.
   c. According to your answer for part b), should the company spend more or less on advertising? Why?

11. Let \( f(4) = 3 \), \( f'(4) = 0 \), and \( f''(4) = 8 \). Then at \( x = 4 \), \( f(x) \) must have a(n) ?.

   a. local minimum
   b. local maximum
   c. global minimum
   d. global maximum
   e. inflection point
12. For the function $f(x) = 3x^4 + 4x^3 - 12x^2 + 2$, find the $x$-value that gives the absolute (global) maximum on the interval $[-1, 1]$.

Let $f(x)$ and $g(x)$ be two functions; values are given in the table below for these functions and their derivatives. Use the information in the table to answer the 3 questions that follow.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x)$</th>
<th>$g(x)$</th>
<th>$f'(x)$</th>
<th>$g'(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>-1</td>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>1</td>
<td>$-1$</td>
<td>2</td>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>3</td>
<td>6</td>
<td>0.5</td>
</tr>
</tbody>
</table>

13. If $h(x) = [g(x)]^3$, then find $h'(0)$.

14. If $k(x) = e^{f(x)}$, then find $k'(1)$.

15. If $p(x) = \frac{g(x)}{f(x)}$, then find $p'(2)$.

16. Given $y = \ln(x^2 + 1)$,
   a. Use its derivative to find the absolute maximum and minimum on $[-1, 2]$.
   b. Find any inflection points of the function $y = \ln(x^2 + 1)$

$$f(x) = \frac{2x^2 + 8}{x}, \quad f'(x) = \frac{2(x-2)(x+2)}{x^3}, \quad \text{and} \quad f''(x) = \frac{15}{x^3}$$

17. Given
   a. Find the $x$ and $y$ intercepts of $f(x)$.
   b. Find the intervals where $f(x)$ is increasing and decreasing; Identify any local maxima and minima.
   c. Find the intervals where $f(x)$ is concave up and concave down. Identify any inflection points, if they exist.
   d. Sketch the graph of $f(x)$.

18. Find the constants $a$ and $b$ in the function $f(x) = ax^3 + bx^2$ if:
   a. the point $(1, -4)$ is an inflection point of $f(x)$.
   b. the point $(-2, -8)$ is a local minimum of $f(x)$. 
19. Let \( f(v) \) be the gas consumption (in liters/km) of a car going at velocity \( v \) (in km/hr). In other words, \( f(v) \) tells you how many liters of gas the car uses to go one kilometer if it is going at velocity \( v \). It is known that \( f(75) = 0.04 \) and \( f'(75) = 0.0004 \).

   a. Let \( g(v) \) be the distance the same car goes on one liter of gas at velocity \( v \). Write \( g(v) \) in terms of \( f(v) \).

   b. Find \( g'(75) \). Interpret your answer.

   c. Find \( g''(75) \). Interpret your answer.

20. True/False

   a. The 8th derivative of \( f(x) = x^8 \) is zero.

   b. If \( f \) is always decreasing and concave down, then \( f \) must have at least one root.

   c. If \( f''(x) = 0 \) then \( f(x) \) has an inflection value at \( x = 5 \).

   d. It is possible that: \( f' > 0 \) everywhere, \( f'' > 0 \) everywhere, and \( f''' < 0 \) everywhere.

   e. It is possible that: \( f' > 0 \) everywhere, \( f'' < 0 \) everywhere, and \( f''' > 0 \) everywhere.

Solutions to Exam III "Sample Questions"

7. \( f'(2) = 15 \), local maximum and \( f'(4) = 12 \), local minimum.
8. a. \( f'(x) = 5 \left( \frac{1}{x} + 2x \right) \left( \frac{1}{2} x^{-2} + 2 \right) \)

   b. \( f'(x) = 2 \sin x + 3 \cos(4x^2 + 1)(2x) \)

   c. \( f'(x) = -9x^2 + 3 \)

   d. \( f'(x) = e^{x^{-1}} - e^{-x} + 6 \cdot 4^{-x} \cdot \log 4 \)

   e. \( f'(x) = -x^2 - \left( x^2 \cdot \frac{10x}{x^2} + 2x \cdot \ln(5x^2) \right) + x \)
9. a. critical values: $x = 0$ and $x = 3$
   
   b. decreasing on $(-\infty, 0) \cup (3, \infty)$; increasing on $(0, 3)$; local minimum when $x = 3$; no local max.
   
   c. concave up on $(-\infty, 0) \cup (3, \infty)$; concave down on $(0, 3)$; Inflection points when $x = 0$ and $x = 2$
   
   d. see graph below. 3 points should be labeled: IP at $(0, 10)$, IP at $(2, -6)$, and local min at $(3, -17)$
   
   e. no
   
   f. Yes, at $(3, -17)$
   
   g. Yes, at $(2, -6)$

![Graph](image)

10. a. $1,620,000$
   
   b. $S'(a) = -100$. When $40,000$ is spent on advertising, each additional $1000$ spent on advertising results in a $100,000$ loss in revenue.
   
   (remember the units are in thousands, so $\frac{dS}{da} = \frac{-100}{1}$. Each additional 1 thousand dollars spent on ads results in a 100 thousand dollar loss in revenue)
   
   c. Spend less (rate is negative, so revenue is decreasing). Draw graph of $S(a)$ to see where max occurs: $(30, 4500)$

11. A
12. $x = 0$
13. 15
14. $\frac{4}{e}$
15. -1
16.  
   a. abs max at (2, 1.6094) and abs min at (0, 0)  
      
   b. IPs at (1, 0.693) and (-1, 0.693)  

17.  
   a. no intercepts  
   b. increasing on (-\infty, 2) \cup (2, \infty) and decreasing on (-2, 0) \cup (0, 2); local min at (2, 8) and local max at (-2, -8).  
   c. concave up on (0, \infty) and concave down on (-\infty, 0). No inflection points (zero not in the domain of f)  
   d. graph

18.  
   a. a = 2 and b = -6  
   b. a = -2 and b = -6  

19.  
   a. \( g(v) = \frac{1}{f(v)} \)  (the units of g are km/liters and the units of f are liters/km)  
   b. 25 km/liter. At a velocity of 75 km/hr, car goes 25 km on one liter of gas.  
   c. -0.25 km/liter per km/hour (or -0.25 hr/liter). At a velocity of 75 km/hr, the fuel efficiency, \( g(v) \), is decreasing at 0.25 km/liter for every additional 1 km/hr you drive. In other words, as your velocity increases, your fuel efficiency decreases.  

20.  
   a. F  
   b. F  
   c. F  
   d. F  
   e. T