```
In [4]:  from sympy import *
         from sympy.plotting import (plot, plot_parametric)
```

CURVE SKETCHING TECHNIQUES IN PYTHON

Determining where a function is increasing/decreasing/concave up/concave down in Python is the same process as done by hand-only without all the intense algebra! Recall said process:

1) Find the first derivative (Key Python command: **diff**)

2) Set = 0 and solve to find the critical values (paying attention to any places where the derivative does not exist as well! Key Python command: **solve**)

3) Numerically test values around the critical values by substituting into the first derivative to determine the sign of the result and where the original function is increasing/decreasing (Key Python command: **subs**). NOTE that with Python, you may be able to graphically determine the signs of the derivative as well (Key Python command: **plot**), though numerical testing is safer.

4) Repeat steps 1) - 3) using the second derivative to determine concavity.

NOTE: The example here (and in future overviews) is NOT a copy/paste to solve the problems in lab. However, it will USE many of the features you will use to solve your problems.
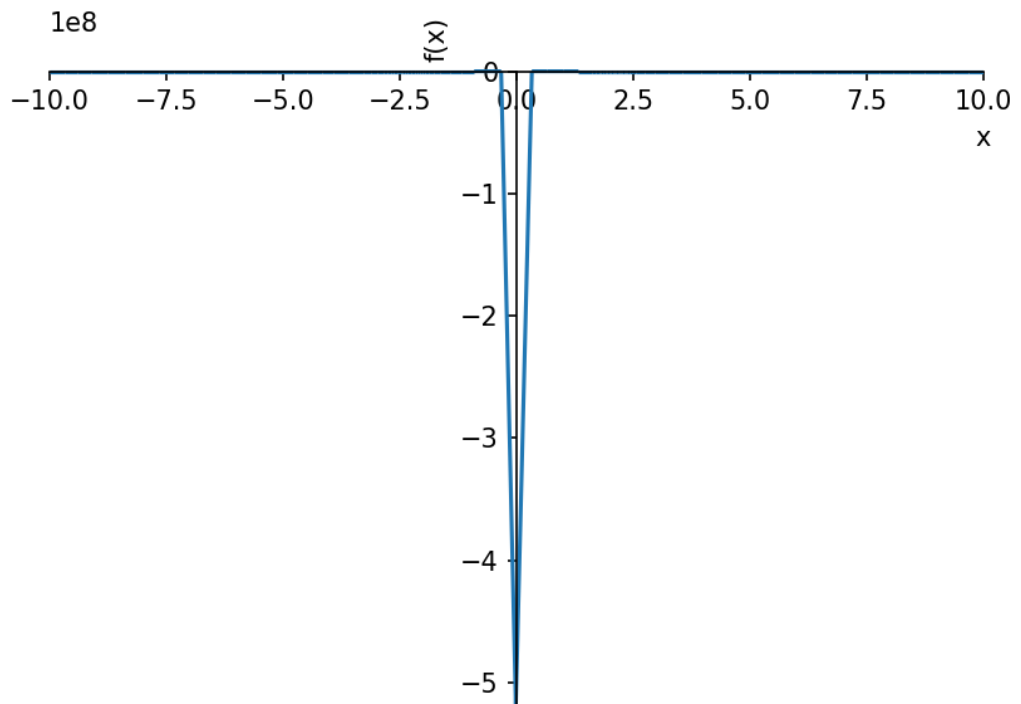
EXAMPLE:

Find the intervals of increase/decrease/concave up/concave down for f(x) = ((x+4)(x-3)^2)/(x^4(x-1))

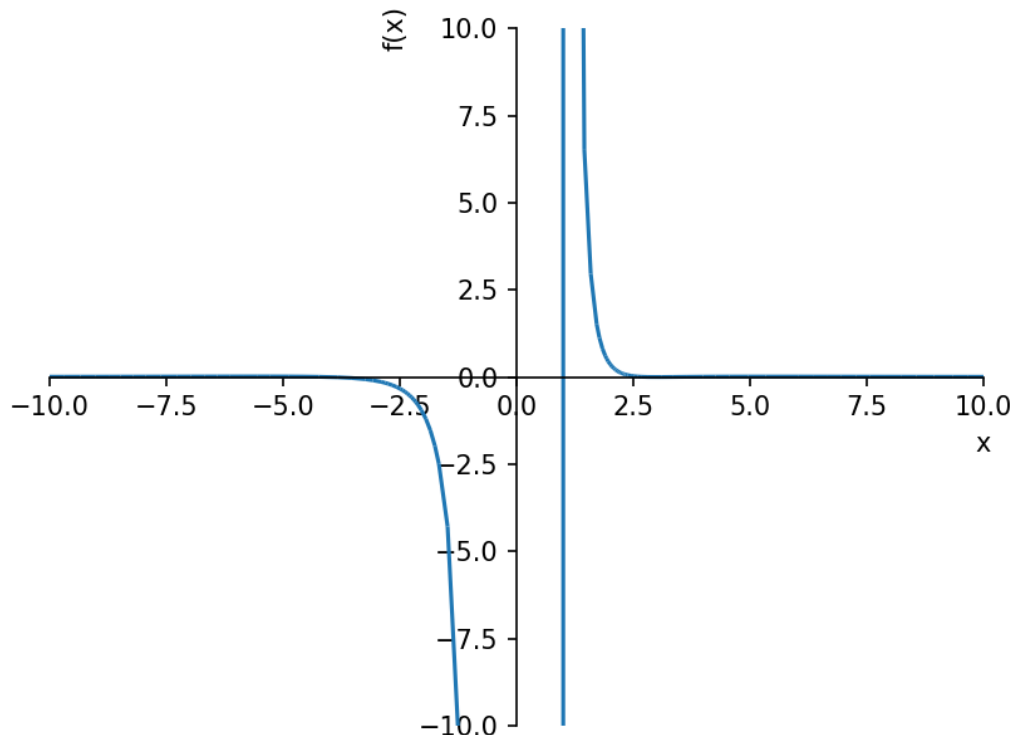So if we have these fancy programs which graph, why not just graph it and see, right?

```
In [5]:  matplotlib notebook
```

In [6]:
```python
x=symbols('x')
f=((x+4)*(x-3)**2)/(x**4*(x-1))
plot(f,(x,-10,10))   #Use the standard -10 to 10 domain with default range
```

1e8

f(x)

| -10.0 | -7.5 | -5.0 | -2.5 | 0 | 2.5 | 5.0 | 7.5 | 10.0 |

x

-1

-2

-3

-4

-5

Out[6]: <sympy.plotting.plot.Plot at 0x1b81812a5f8>

In [7]:
```python
# That didn't seem to help.  How about the default -10 to 10 range as well
plot(f,(x,-10,10),ylim=[-10,10])
```


Out[7]: `<sympy.plotting.plot.Plot at 0x1b8190dcdd8>`

So it looks like the function is always decreasing and concave down, then concave up. Of course, you may notice that the function has vertical asymptotes at x=0 and x=1. With this information, we can guess that f is decreasing and concave down (-oo, 0) and decreasing and concave up (1, oo), but we can't see anything on the interval (0, 1). We could keep playing around with different windows until we think we know everything, but that could take just as long as the calculus (and may also not be accurate as we shall see!)

In [8]:
```python
# Step 1: derivative
df=diff(f,x)
print('The derivative of f is',df.simplify()) # Let's see what it looks lik
e simplified
print('Or in expanded form, ',df.expand()) # Now expanded...not very helpfu
l
```

```
The derivative of f is (-2*x**4 + 7*x**3 + 56*x**2 - 225*x + 144)/(x**5*(x*
*2 - 2*x + 1))
Or in expanded form,  -x**3/(x**6 - 2*x**5 + x**4) - 4*x**3/(x**6 - x**5) +
2*x**2/(x**6 - 2*x**5 + x**4) + 8*x**2/(x**6 - x**5) + 3*x**2/(x**5 - x**4)
+ 15*x/(x**6 - 2*x**5 + x**4) + 60*x/(x**6 - x**5) - 4*x/(x**5 - x**4) - 3
6/(x**6 - 2*x**5 + x**4) - 144/(x**6 - x**5) - 15/(x**5 - x**4)
```

```
In [10]:  # Step 2: solve df=0
          cvals=solve(df,x)
          print(cvals) # This lets us see the list of critical values.  We get 4 solu
          tions, but 3 of them appear to be complex
          cvals_float=[i.evalf() for i in cvals] # Use list comprehension to convert
           them all to decimals
          print('The critical values are',cvals_float)
```

```
[3, 1/6 - 355/(12*(-1/2 - sqrt(3)*I/2)*(515/2 + 15*sqrt(179979)*I/8)**(1/
3)) - (-1/2 - sqrt(3)*I/2)*(515/2 + 15*sqrt(179979)*I/8)**(1/3)/3, 1/6 - (-
1/2 + sqrt(3)*I/2)*(515/2 + 15*sqrt(179979)*I/8)**(1/3)/3 - 355/(12*(-1/2 +
sqrt(3)*I/2)*(515/2 + 15*sqrt(179979)*I/8)**(1/3)), 1/6 - (515/2 + 15*sqrt
(179979)*I/8)**(1/3)/3 - 355/(12*(515/2 + 15*sqrt(179979)*I/8)**(1/3))]
The critical values are [3.00000000000000, 0.820889295432331 + 0.e-22*I, 5.
24902185992962 - 0.e-22*I, -5.56991115536196 + 0.e-20*I]
```

So we see that, although some of the solutions LOOK complex, the imaginary parts are all infinitessimal (the "e-22" is scientific notation, meaning x 10^-22!) We also see that 0 and 1 are vertical asymptotes, so there could be changes in direction there as well. We now analyze the sign of the derivative around each critical value (to 2 decimal places, the critical values are [-5.57, 0, 0.82, 1, 3, 5.25]) We'll try a graphical analysis of f' first
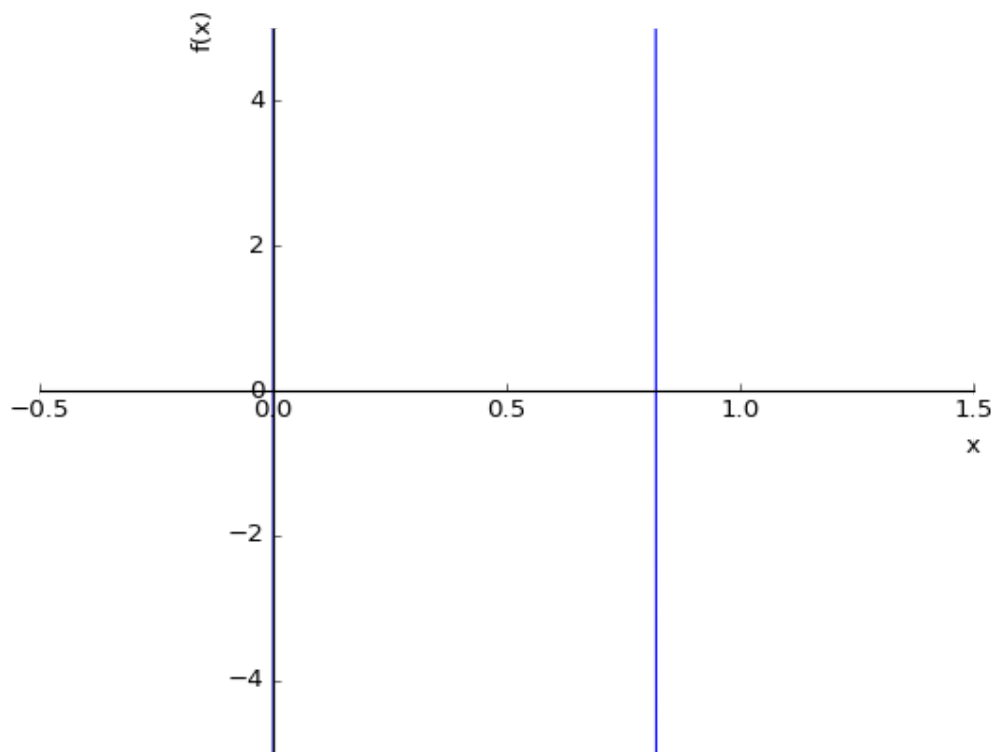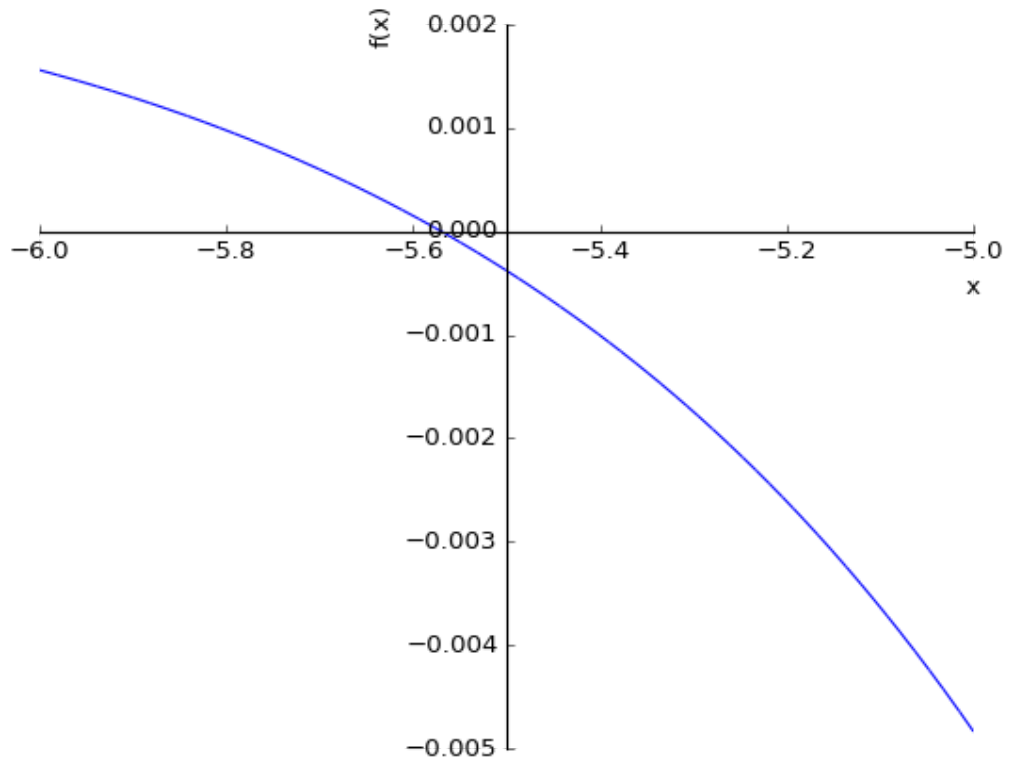
In [11]: `plot(df,(x,-6,5.5),ylim=[-5,5])`
`# choosing a domain which just encloses our critical values and a small y-r`
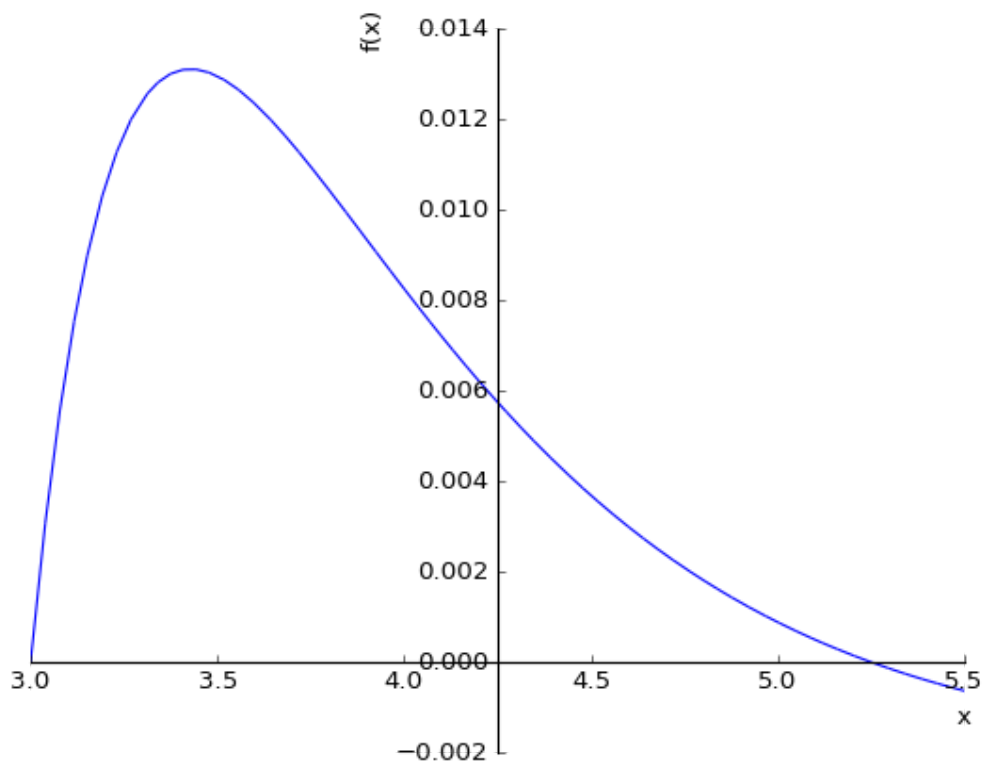`ange to view the signs.`
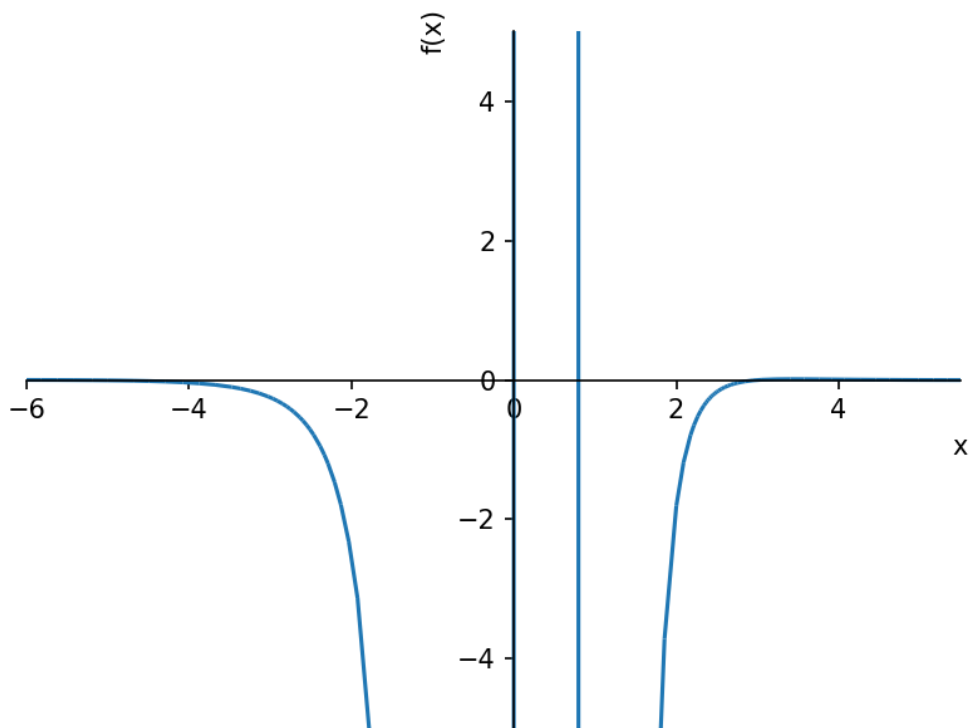
Out[11]: `<sympy.plotting.plot.Plot at 0x1b8190b5320>`

Clearly, f' is negative from (-5.57,0) and (1,3), but the others aren't so obvious. So let's zoom in around each critical value

In [8]:
```
plot(df,(x,-6,-5))
plot(df,(x,-0.5,1.5),ylim=[-5,5])
plot(df,(x,3,5.5))
```

Out[8]: <sympy.plotting.plot.Plot at 0x7f235bde53c8>

We now know f' > 0 on (-oo,-5.57)u(3,5.25) and f' < 0 on (-5.57,0)u(1,3)u(5.25,oo), but it's still not obvious what is happening between 0 and 1. We could keep trying graphs, but numerical testing is easier...ESPECIALLY with list comprehension!

```
In [9]:  testvals=[-6,-3,0.4,0.9,2,4,6] # Choosing numbers around the critical value
         s [-5.57, 0, 0.82, 1, 3, 5.25]
         dfsign=[df.subs(x,i) for i in testvals]
         print('The derivatives at the test values are',dfsign)
         print('So f is increasing on (-00,-5.57),(0,0.82), and (3,5.25)')
         print('and f is decreasing on (-5.57,0),(0.82,1),(1,3), and (5.25,oo)')
         print('Which is nothing like our intial guess!')
```

```
The derivatives at the test values are [11/7056, -1/4, 17186.6319444444, -1
583.29522938577, -29/16, 19/2304, -1/720]
So f is increasing on (-00,-5.57),(0,0.82), and (3,5.25)
and f is decreasing on (-5.57,0),(0.82,1),(1,3), and (5.25,oo)
Which is nothing like our intial guess!
```

In [13]:
```python
# Now repeat with second derivative
ddf=diff(f,x,2)
print('The second derivative is',ddf.simplify())
c2vals=solve(ddf,x)
print(c2vals)
c2vals_float=[i.evalf() for i in c2vals]
print('The 2nd derivative critical values are',c2vals_float)
# Ran it here to see we DO get two complex numbers, so we ignore those
```

```
The second derivative is 2*(3*x**5 - 15*x**4 - 133*x**3 + 759*x**2 - 954*x
+ 360)/(x**6*(x**3 - 3*x**2 + 3*x - 1))
[CRootOf(3*x**5 - 15*x**4 - 133*x**3 + 759*x**2 - 954*x + 360, 0), CRootOf
(3*x**5 - 15*x**4 - 133*x**3 + 759*x**2 - 954*x + 360, 1), CRootOf(3*x**5 -
15*x**4 - 133*x**3 + 759*x**2 - 954*x + 360, 2), CRootOf(3*x**5 - 15*x**4 -
133*x**3 + 759*x**2 - 954*x + 360, 3), CRootOf(3*x**5 - 15*x**4 - 133*x**3
+ 759*x**2 - 954*x + 360, 4)]
The 2nd derivative critical values are [-7.12681097468148, 3.4279437529788
5, 7.05877048468240, 0.820048368510115 - 0.152914656877877*I, 0.82004836851
0115 + 0.152914656877877*I]
```

In [14]:
```python
testvals=[-8,-1,0.5,2,5,10] # Remember critical values are [-7.13, 0, 1, 3.
43, 7.06]
ddfsign=[ddf.subs({x:i}) for i in testvals]
print('The second derivatives at the test points are',ddfsign)
print('So f is concave up (-oo,-7.13)u(1,3.43)u(7.06,oo)')
print('and f is concave down (-7.13,0)u(0,1)u(3.43,7.06)')
```

```
The second derivatives at the test points are [4385/11943936, -547, -56608.
0000000000, 35/4, -103/25000, 2093/9112500]
So f is concave up (-oo,-7.13)u(1,3.43)u(7.06,oo)
and f is concave down (-7.13,0)u(0,1)u(3.43,7.06)
```

So our initial guess of NO change in direction and only ONE change in concavity is actually FIVE changes in direction and FOUR changes in concavity!

In [ ]: