

Group Member Names:

```
In [1]: from sympy import *  
        from sympy.plotting import plot, plot_parametric
```

## Lab 4 Template

Each part of each problem should be solved in its own cell.

### Question 1

Let  $f(x) = xe^{-|x|}$ .

a.) Plot  $f$  on  $[-5, 5]$ .

b.) Find the equations of all horizontal tangent lines. (Note: when declaring  $x$ , be sure to use **symbols('x', real=True)**)

In [ ]:

In [ ]:

### Question 2

Let  $f(x) = x \sin(x)$ .

a.) Find the first 16 derivatives of  $f$ .

b.) Given a number  $n$  that is divisible by 4, in a separate print statement for each answer, state what  $f^{(n)}(x)$ ,  $f^{(n+1)}(x)$ ,  $f^{(n+2)}(x)$ , and  $f^{(n+3)}$  are.

In [ ]:

```
In [2]: print("f^(n)(x)      = ")  
        print("f^(n + 1)(x) = ")  
        print("f^(n + 2)(x) = ")  
        print("f^(n + 3)(x) = ")
```

```
f^(n)(x)      =  
f^(n + 1)(x) =  
f^(n + 2)(x) =  
f^(n + 3)(x) =
```

### Question 3

This problem will explore the properties of derivatives in approximating functions with polynomials.

a.) Plot the function  $f(x) = e^{-x^2}$  on  $[-5, 5]$  with **ylim=[-1, 1]** and print out  $f(0)$ .

b.) Compute value of the first 8 derivatives of  $f(x)$  at  $x = 0$ . (Only print out the evaluations)

c.) A Taylor polynomial is a polynomial whose coefficients are chosen using the derivatives of another function. The general formula for the  $n - th$  Taylor polynomial centered at zero is given by  $T_n(x) = f(0) + \frac{f'(0)}{1!}x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \dots + \frac{f^{(n-1)}(0)}{(n-1)!}x^{n-1} + \frac{f^{(n)}(0)}{(n)!}x^n$ , where ! is the **factorial** function from the previous assignment. Plot  $T_2(x)$ ,  $T_4(x)$ ,  $T_6(x)$ , and  $T_8(x)$  on  $[-5, 5]$  on the same axes with **ylim=[-1, 1]**. (Hint: using list comprehension and the python **sum** command on a list can greatly reduce the amount of typing necessary for this problem)

d.) Evaluate the absolute error given by  $|T_n(x) - f(x)|$  for  $n = 2, 4, 6, 8$  at  $x = 1$  and print the output in four separate print statements, making sure to use **.evalf()** to get a decimal representation.

In [ ]:

In [ ]:

In [ ]:

In [3]:

```
print("Error for T_2 is:", )  
print("Error for T_4 is:", )  
print("Error for T_6 is:", )  
print("Error for T_8 is:", )
```

```
Error for T_2 is:  
Error for T_4 is:  
Error for T_6 is:  
Error for T_8 is:
```