

MATH 151

FALL 1996

EXAM III-A

Part I - Multiple Choice

For problems 1 through 11, mark the correct answer on your scantron sheet in PENCIL. Each question is worth 10 points (201 points total for the exam). No calculators.

1. Compute $\lim_{x \rightarrow 0} \frac{\cos(x) - 1}{x^2}$

- a) 0 b) 1 c) $-\frac{1}{2}$ d) -2 e) ∞

2. Differentiate $\tan^{-1}(x^2)$

- a) $\frac{2x}{x^4 + 1}$ b) $2x \sec^{-2}(x^2)$ c) $-\tan^{-2}(x^2)$ d) $\left(\frac{2x}{x^2 + 1}\right)^2$ e) $\cot(x^2)$

3. Find the absolute maximum and minimum values of the function $f(x) = x^3 - 3x + 1$ on the interval $[-1, 3]$.

- a) minimum f -value is 3; the maximum f -value is 19
b) minimum f -value is -1 ; the maximum f -value is 3
c) minimum f -value is -4 ; the maximum f -value is 20
d) minimum f -value is -1 ; the maximum f -value is 19
e) minimum f -value is -8 ; the maximum f -value is 10

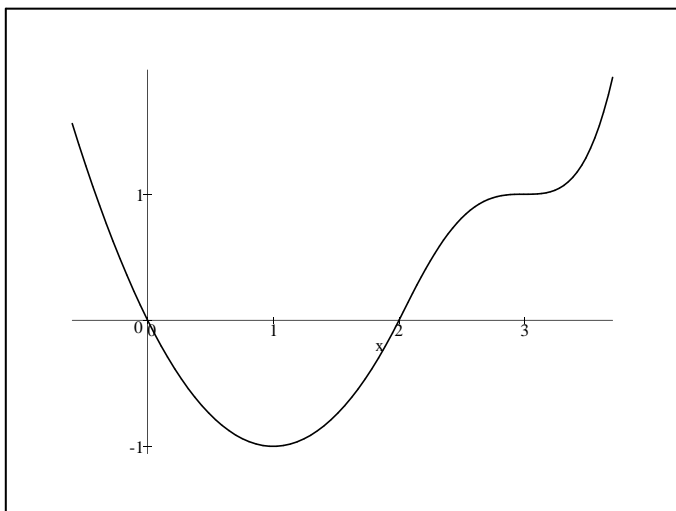
4. Find the number of real solutions to the equation $x^3 - 3x + 3 = 0$.

- a) none b) one c) two d) three e) four

5. Suppose that f is a differentiable function with $f(0) = 2$ and $f'(x) \leq 3$ for all x . What is the largest possible value for $f(2)$?

- a) 5 b) 6 c) 10 d) 8 e) 4

6. The graph of the *derivative*, f' , of a function f is given below. Which x value(s) correspond to a local minimum for the function f ?



- a) $x = 0$ b) $x = 1$ c) $x = 2$ d) $x = 0$ and $x = 2$ e) $x = 1$ and $x = 3$

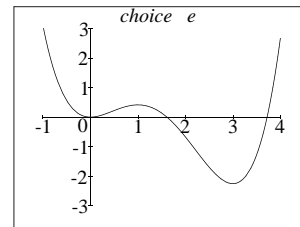
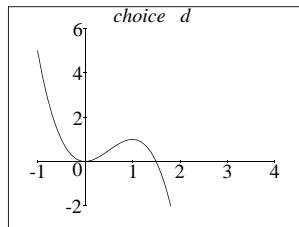
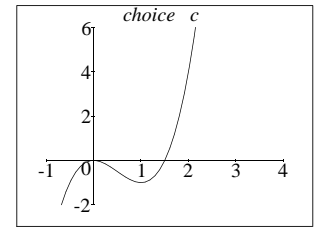
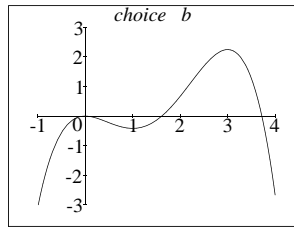
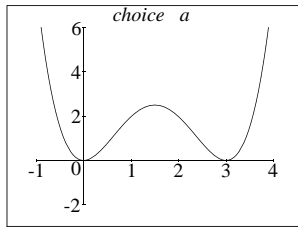
7. Referring to the above graph of f' , which x value(s) correspond to a point of inflection for the function f ?

- a) $x = 0$ b) $x = 1$ c) $x = 3$ d) $x = 0$ and $x = 2$ e) $x = 1$ and $x = 3$

8. Which of the following statements is true about the graph of the function $f(x) = xe^x$?

- a) The graph is increasing for $x > -1$ and concave down for $x < 0$.
- b) The graph is increasing for $x > -2$ and concave down for $x < -1$.
- c) The graph is increasing for all x and concave down for all x .
- d) The graph is increasing for $x > -1$ and concave down for $x < -2$.
- e) The graph is increasing for all x and concave up for all x .

9. Which of the following figures represents the graph of the function $f(x) = \frac{x^4}{4} - \frac{4x^3}{3} + \frac{3x^2}{2}$?



10. If $f'(x) = 3 \cos(x) + 5 \sin(x)$ with $f(0) = 4$, then find $f(x)$.

- a) $f(x) = 3 \sin x - 5 \cos x + 9$ b) $f(x) = 3 \cos x + 5 \sin x + 1$ c) $f(x) = -3 \sin x + 5 \cos x - 1$
 d) $f(x) = -3 \sin x + 5 \cos x - 1$ e) $f(x) = 3 \sin x - 5 \cos x + 4$

11. Suppose $\int_1^5 f(x) dx = 7$, $\int_3^8 f(x) dx = 6$ and $\int_3^5 f(x) dx = 2$. What is $\int_1^8 f(x) dx$?

- a) 13 b) 15 c) 3 d) 1 e) 11

Part II - Work Out Problems

Work the following problems in the space provided. Clearly, show all your steps for full credit. Each problem is worth 13 points. Calculators are allowed on this part.

12. If a sample of radon has decayed to 60% of its original amount in 5 days, then what is its half-life (the time required to decay to 50% of its original amount)?

13. Compute the *exact* value of the limit $\lim_{x \rightarrow 0} (1 + 2 \sin x)^{\frac{1}{x}}$. Show all your steps.

14. By differentiating $\cos y = x$ implicitly, show that $\frac{d}{dx} [\cos^{-1}(x)] = \frac{-1}{\sqrt{1-x^2}}$ for $-1 < x < 1$.
15. A box is to be constructed so that its base is a rectangle whose length is twice as long as its width. The volume of the box is 36 cubic inches and the box has no top. Find the dimensions of this box that minimizes the amount of material used.

16. A projectile is shot from a cannon with an initial muzzle velocity of 500 meters per second. If the angle of elevation of the cannon is $30^\circ = \pi/6$ radians, then derive the coordinates of the projectile relative to the cannon t seconds after it is fired. Ignore air resistance and note that the gravitational constant is $g = -9.8$ (in meters per second per second). Do NOT just quote a formula for your answer. Show all your steps and derive any formulas that you use.
17. A car braked with a constant deceleration of 20 ft /sec^2 (i.e acceleration = -20) over a distance of 160 feet before coming to a stop. How fast was the car traveling when the brakes were first applied?

18. Approximate the integral $\int_1^3 \sqrt{1+x^2} dx$ by using the midpoint rule with a partition of four equal intervals. You need not simplify your answer.