

**MATH 151: Spring 1996**  
**Exam III: Test Form A**

**Part I. MULTIPLE CHOICE, NO PART CREDIT, NO CALCULATORS**

The SCANTRON forms will be collected at the end of 1 hour.

(5 points each)

1. Differentiate  $y = xe^{-x^2}$ .

- (a)  $y' = \frac{1}{2xe^{x^2}}$       (b)  $y' = \frac{e^{x^2} - x(x^2e^{x^2-1})}{(e^{x^2})^2}$       (c)  $y' = e^{-x^2} + 2xe^{-x^2}$       (d)  $y' = e^{-x^2} - 2x^2e^{-x^2}$   
(e)  $y' = -2xe^{-x^2}$

2. After the substitution  $u = x^2 - 1$  the integral  $\int_2^4 \frac{x}{\sqrt{x^2 - 1}} dx$  becomes

- (a)  $\int_3^{15} \frac{1}{2\sqrt{u}} du$       (b)  $\int_3^{15} \frac{2}{\sqrt{u}} du$       (c)  $\int_3^{15} \frac{u+1}{\sqrt{u}} du$       (d)  $\int_2^4 \frac{1}{\sqrt{u}} du$       (e)  $\int_2^4 \frac{u}{\sqrt{u^2 - 1}} du$

3. Which of the following is an antiderivative of  $f(x) = x - \frac{1}{x^3}$ ?

- (a)  $\frac{x^2}{2} - \frac{2}{x^2}$       (b)  $\frac{x^2}{2} + \frac{1}{2x^2}$       (c)  $1 + \frac{3}{x^4}$       (d)  $1 - \frac{3}{x^3}$       (e)  $1 + \frac{3}{x^2}$

4. If  $f(x) = \int_1^{2x-1} e^{t^2} dt$ , then  $f'(0) =$

- (a)  $-\frac{2}{e}$       (b)  $\frac{1}{e}$       (c)  $2e$       (d)  $e$       (e)  $e^2 - 1$

5. Solve the equation  $\ln(x+1) + \ln(x-1) = 0$ , for  $x$ .

- (a)  $x = 0$       (b)  $x = 1$       (c)  $x = \pm 1$       (d)  $x = \sqrt{2}$       (e)  $x = \pm\sqrt{2}$

6. Evaluate  $\int_{-e^2}^{-e} \frac{1}{x} dx$ .

- (a) 1      (b) -1      (c)  $\frac{1}{e^2} - \frac{1}{e}$       (d)  $\frac{1}{e} - \frac{1}{e^2}$       (e) Undefined

7. Find the inverse function of  $f(x) = \frac{1+x}{3-2x}$ .

- (a)  $f^{-1}(x) = \frac{1+x}{3-x/2}$       (b)  $f^{-1}(x) = \frac{3-2x}{1+x}$       (c)  $f^{-1}(x) = \frac{1+1/x}{3-2/x}$       (d)  $f^{-1}(x) = \frac{1-x}{3+2x}$       (e)  $f^{-1}(x) = \frac{3x-1}{2x+1}$

8. Which of the following properties of  $f(x)$  guarantees that the right Riemann sum of  $f(x)$  on the interval  $[a, b]$  (that is the Riemann sum using right endpoints) is less than  $\int_a^b f(x) dx$ , for any partition of  $[a, b]$ ?

- (a)  $f(x)$  is concave up.      (b)  $f(x)$  is concave down.      (c)  $f(x)$  is continuous.      (d)  $f(x)$  is increasing.      (e)  $f(x)$  is decreasing.

9. The acceleration of a car is given by  $a(t) = 3t + 2$  ft/sec<sup>2</sup>. If the car is at rest at time  $t = 0$ , what is its velocity at time  $t = 2$ ?

- (a) 8 ft/sec (b) 10 ft/sec (c) 12 ft/sec (d)  $\frac{9}{2}$  ft/sec (e)  $\frac{13}{4}$  ft/sec

10. An antiderivative of  $xe^x$  is

- (a)  $\frac{x^2}{2}e^x$  (b)  $xe^x + e^x$  (c)  $xe^x - e^x$  (d)  $x + \ln x$  (e)  $xe^x$

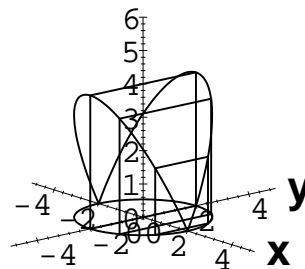
11. Evaluate  $\int \frac{dx}{x \ln x}$ .

- (a)  $\ln |\ln x| + C$  (b)  $\ln |x|$  (c)  $\ln |x| + C$  (d)  $\ln |x \ln x| + C$  (e)  $(\ln x) \ln |\ln x| + C$

**Part II. WORK OUT PROBLEMS,** PART CREDIT will be given. CALCULATORS ARE PERMITTED after the SCANTRONS are collected. Show all relevant steps in your solution. Clearly indicate your answer. Unsupported answers will not be given credit. Only work shown in the space provided will be graded.

12. (8 points) Find  $f(x)$  if  $f''(x) = 12x^2 - e^{4x}$ ,  $f(0) = 1$  and  $f'(0) = 0$ .

13. (7 points) Find the volume of the solid whose base is the circle  $x^2 + y^2 = 4$  and whose cross sections perpendicular to the  $x$ -axis are squares.



14. (7 points) Find the volume of the solid obtained by rotating about the  $x$ -axis the region bounded by the curve  $y = \sqrt{x-1}$ , and the lines  $x = 2$ ,  $x = 5$  and  $y = 0$ .

15. (8 points) (a) Compute  $\frac{1}{4} \sum_{i=1}^4 \sqrt{\frac{i}{4}}$ .

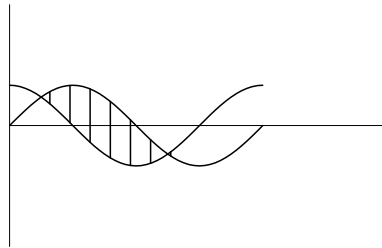
(b) The quantity  $\frac{1}{n} \sum_{i=1}^n \sqrt{\frac{i}{n}}$  is a Riemann sum for which of the following integrals?

(Circle your answer; no explanation is required.)

$$\int_1^n \sqrt{\frac{1}{x}} dx \quad \int_0^1 \frac{1}{\sqrt{x}} dx \quad \int_0^1 \sqrt{x} dx \quad \frac{1}{n} \int_{1/n}^1 \sqrt{\frac{1}{x}} dx \quad \frac{1}{n} \int_{1/n}^1 \sqrt{x} dx$$

(c) Use your answer to (b) to compute  $\lim_{n \rightarrow \infty} \frac{1}{n} \sum_{i=1}^n \sqrt{\frac{i}{n}}$ .

16. (7 points) Find the area of the shaded region between the graphs of  $y = \sin x$  and  $y = \cos x$  as shown in the plot below.



17. (8 points) Compute  $\int_0^2 x e^{x^2} dx$ .