MATH 151, FALL SEMESTER 2004
COMMON EXAMINATION I - VERSION A

Name (print):______________________________________

Signature: ______________________________________

Instructor's name: ______________________________________

Section No: ______________________________________

Seat No: ______________________________________

INSTRUCTIONS

1. In Part 1 (Problems 1–13), mark the correct choice on your ScanTron form using a No. 2 pencil. *For your own record, also mark your choices on the exam.* ScanTrons will be collected from all examinees after one hour, and will *not* be returned.

2. Calculators may *not* be used in Part 1. The use of calculators is permitted *only after the first hour has elapsed and all ScanTrons have been collected*.

3. In Part 2 (Problems 14–18), present your solutions in the space provided. **Show all your work** neatly and concisely, and **indicate your final answer clearly**. You will be graded, not merely on the final answer, but also on the quality and correctness of the work leading up to it.

4. Be sure to **write your name, section number, and version letter of the exam on the ScanTron form**.

QN	PTS
1–13	
14	
15	
16	
17	
18	
TOTAL	

Part 1 – Multiple Choice (52 points)

Read each question carefully; each problem is worth **4 points**. *Calcluators are* **not** *allowed for this part of the exam.*

1. If $\mathbf{a} = \langle 2, 3\rangle$ and $\mathbf{b} = \langle 1, -1\rangle$, find $2\mathbf{a} - 3\mathbf{b}$.

(a) $\langle 1, 9\rangle$

(b) $\langle 4, 11\rangle$

(c) $\langle 1, 3\rangle$

(d) $\langle 4, 7\rangle$

(e) $\langle 1, 1\rangle$

2. Let A, B, and C be the vertices of a triangle. If $\overrightarrow{AB} = \mathbf{a}$ and $\overrightarrow{AC} = \mathbf{b}$, what is $\overrightarrow{BC}$?

(a) $\mathbf{a} + \mathbf{b}$

(b) $\mathbf{a} - \mathbf{b}$

(c) $\mathbf{b} - \mathbf{a}$

(d) $\mathbf{a} \cdot \mathbf{b}$

(e) $\frac{1}{2}(\mathbf{a} + \mathbf{b})$

3. Let $\mathbf{a} = \langle -2, 3\rangle$ and $\mathbf{b} = \langle 1, 2\rangle$. Find $\text{comp}_{\mathbf{a}}\mathbf{b}$, *i.e.*, the scalar projection of $\mathbf{b}$ onto $\mathbf{a}$.

(a) $\sqrt{13}/4$

(b) $8/\sqrt{13}$

(c) $\sqrt{13}/8$

(d) $4/\sqrt{13}$

(e) $4/13$

4. Suppose that $\mathbf{u}$ and $\mathbf{v}$ are unit vectors which subtend an angle of 60 degrees between them. Compute the dot product $\mathbf{v} \cdot (2\mathbf{u} - 3\mathbf{v})$.

(a) -2

(b) -1

(c) 0

(d) 1

(e) 2

5. The parametric curve determined by the equations $x(t) = \sin t$, $y(t) = \cos^2 t$, $0 \le t \le \pi/2$, forms a:

(a) part of a parabola

(b) part of a hyperbola

(c) part of a circle

(d) line segment

(e) none of the above

6. Suppose that f is a function defined on an open interval containing the point 2, and that $f(2) = 3$. Which of the following statements is <u>always</u> true?

(a) $\lim_{x\to 2} f(x) = 3$.

(b) Either $\lim_{x\to 2^-} f(x)$ or $\lim_{x\to 2^+} f(x)$ must exist.

(c) If $\lim_{x\to 2} f(x)$ exists, then f is continuous at $x = 2$.

(d) If $\lim_{x\to 2} f(x) = 3$, then f is continuous at $x = 2$.

(e) If $\lim_{x\to 2} f(x) = 3$, then f is differentiable at $x = 2$.

7. Which of the following intervals contains a number c satisfying the equation $c^3 + c - 1 = \pi^2$?

(a) $(0, 1)$

(b) $(1, 2)$

(c) $(2, 3)$

(d) $(3, \infty)$

(e) none of the above

8. Evaluate $\displaystyle\lim_{x\to\infty} \frac{\sqrt{2x^4 + 3x^2 + 1}}{3x^2 + 2x + 4}$.

(a) $2/3$

(b) $\sqrt{3}/2$

(c) $\sqrt{2}/3$

(d) $3/\sqrt{2}$

(e) $1/4$

9. Find all the vertical asymptotes of the curve $y = \dfrac{x-2}{x^2-4}$.

(a) $x = \pm 2$

(b) $x = -2$ only

(c) $x = 2$ only

(d) $x = 0$

(e) there are no vertical asymptotes

10. Let $f(x) = |2x - 3|$. Find the value(s) of x for which $f'(x)$ does not exist.

(a) $x = \pm 3/2$

(b) $x = 2/3$

(c) $x = 3/2$

(d) $x = 0, 3/2$

(e) $x = 0, 2/3$

11. Differentiate the function $f(x) = \dfrac{2x - 1}{x^2 + 1}$ with respect to x.

(a) $\dfrac{2(1 + x - x^2)}{x^2 + 1}$

(b) $\dfrac{2(1 + x - x^2)}{(x^2 + 1)^2}$

(c) $\dfrac{2(x^2 - x - 1)}{(x^2 + 1)^2}$

(d) $\dfrac{1}{x}$

(e) $-\dfrac{2}{x^2}$

12. A particle is moving along a straight line and its position function is given by $s(t) = t^2 + t + 1$, where t is measures in seconds and s is measured in meters. Find the velocity of the particle (in meters per second) after 2 seconds.

(a) 2

(b) 5

(c) 3

(d) 7

(e) 0

13. Suppose that $f(0) = 0$ and $\lim\limits_{x \to 0} \dfrac{f(x)}{x} = -1$. If $g(x) = (2x - 1)f(x)$, find $g'(0)$. (Hint: What is $f'(0)$?)

(a) 1

(b) -1

(c) 0

(d) -2

(e) 2

Part 2 (52 points, including a bonus of 4)

Calculators are allowed for this part of the exam. Refer to the front page for further instructions.

14. (`8 points`) A woman walks due west on the deck of a ship at 3 miles per hour. The ship is sailing north at a speed of 22 miles an hour. Find the speed and direction (expressed as an angle θ) of the woman relative to the surface of the water. Draw a figure and label θ.

15. (`8 points`) Obtain a vector equation of the straight line passing through the point $(3, 2)$ and perpendicular to the vector joining the points $(1, 1)$ and $(2, -1)$.

16. (`9 points`) Let c be a fixed real number and let f be defined as follows:

$$f(x) = \begin{cases} 2c^2x^2 + cx + c, & \text{if } x < 1; \\ 1, & \text{if } x = 1; \\ cx + 1, & \text{if } x > 1. \end{cases}$$

Find all possible values of c for which $\lim_{x \to 1} f(x)$ exists. Explain your reasoning carefully and concisely.

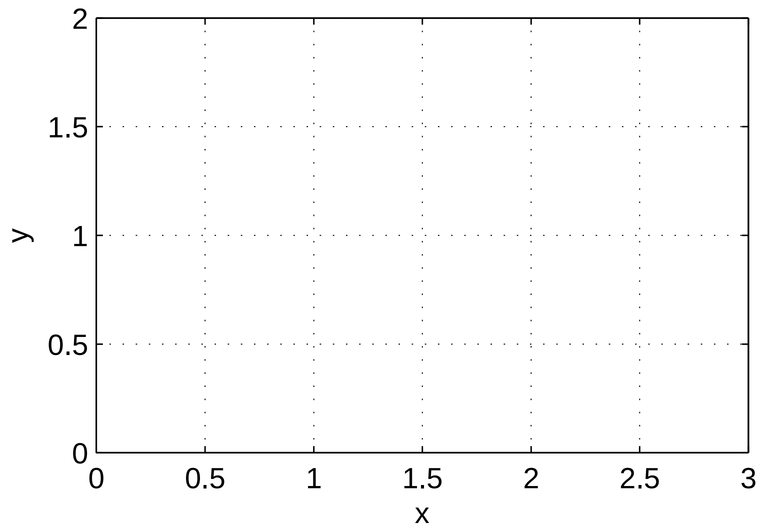
17. (i) (`3 points`) Give a precise definition of the derivative of a function f at a point a.

(ii) (`7 points`) Let $f(x) = \dfrac{1}{2x+3}$. Use the **definition of the derivative** to compute $f'(1)$.
(*Note: No credit will be given for using any other method, correct answer notwithstanding.*)

(iii) (`3 points`) Obtain an equation of the tangent to the curve $y = \dfrac{1}{2x+3}$ when $x = 1$.

18. Consider the functions f and g defined on the closed interval $[0, 3]$ as follows: the graph of f consists of a straight-line segment from the point $(0, 1)$ to the point $(1, 1)$, followed by a line segment from $(1, 1)$ to $(3, 2)$. The graph of g comprises a line segment from the point $(0, 2)$ to the point $(3/2, 0)$, followed by a line segment from $(3/2, 0)$ to $(3, 1)$.

(i) (`2 points`) Sketch the graphs of f and g in the grid provided below.



(ii) (`1 point`) At which value(s) of x in the open interval $(0, 3)$ is f not differentiable?

(iii) (`1 point`) At which value(s) x in the open interval $(0, 3)$ is g not differentiable?

(iv) (`5 points`) Find the derivative of fg at the point $x = 1/2$.

(v) (`5 points`) Find the derivative of $\dfrac{f}{f+g}$ at the point $x = 2$.