Fall 2006 Math 151 Common Exam 1A Thu, 28/Sep/2006

For official use only!

Name (print): ____________________

Signature: ____________________

Instructor: ____________________

Section # ____________________

Seat # ____________________

QN	PTS
1–13	
14	
15	
16	
17	
18	
19	
20	
Total	

Instructions

1. In **Part 1** (Problems 1–13), mark the correct choice on your ScanTron form using a No. 2 pencil. *For your own record, also mark your choices on your exam!* ScanTrons will be collected from all examinees **after 90 minutes** and will *not* be returned.

2. Be sure to write your **name**, **section** number, and **version** of the exam (**1A** or **1B**) on your ScanTron.

3. In **Part 2** (Problems 14–20), present your solutions in the space provided. **Show all your work** neatly and concisely, and **indicate your final answer clearly**. You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.

4. Neither calculators nor computers are permitted on this exam.

5. Please turn off all cell phones so as not to interrupt other students.

Part 1: Multiple Choice (52 points)

Read each question carefully. Each problem in Part 1 is worth **4 points**.

1. Evaluate $\lim_{h \to 0} \dfrac{\sqrt{9+h}-3}{h}$.

   (a) $\dfrac{1}{6}$
   (b) 1
   (c) 0
   (d) ∞
   (e) does not exist

2. Suppose $\mathbf{v}$ and $\mathbf{w}$ are vectors with $|\mathbf{v}| = 2$ and $|\mathbf{w}| = 3$. If the angle between $\mathbf{v}$ and $\mathbf{w}$ is 60 degrees, compute $\mathbf{v} \cdot (2\mathbf{w} - 3\mathbf{v})$.

   (a) $12 - 3\sqrt{2}$
   (b) $6\sqrt{3} - 12$
   (c) 0
   (d) $2\sqrt{13}$
   (e) -6

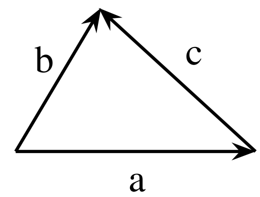
3. Let $\mathbf{a} = 2\mathbf{i} - 3\mathbf{j}$ and $\mathbf{b} = \mathbf{i} + 2\mathbf{j}$. Find $\text{comp}_{\mathbf{a}}\mathbf{b}$, the scalar projection of $\mathbf{b}$ onto $\mathbf{a}$.

   (a) $-\dfrac{4\sqrt{5}}{5}$
   (b) $-\dfrac{4\sqrt{13}}{13}$
   (c) -4
   (d) $\dfrac{4\sqrt{5}}{5}$
   (e) $\dfrac{4\sqrt{13}}{13}$

4. The following statements pertain to the figure below. Which statements are true?

(i) $\mathbf{a}+\mathbf{b}=\mathbf{c}$ (ii) $\mathbf{a}-\mathbf{b}=\mathbf{c}$ (iii) $\mathbf{b}-\mathbf{a}=\mathbf{c}$ (iv) $|\mathbf{b}|+|\mathbf{c}|=|\mathbf{a}|$

(a) Only statement (i) is true.
(b) Only statement (ii) is true.
(c) Only statement (iii) is true.
(d) Statements (iii) and (iv) are both true.
(e) Statements (ii) and (iii) are both true.



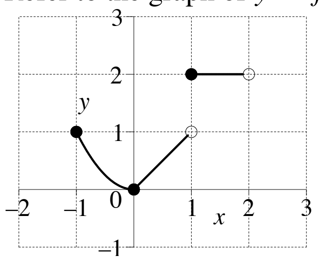
5. If $\mathbf{a}=\langle 1,-4\rangle$ and $\mathbf{b}=\langle 2,3\rangle$, find $3\mathbf{a}-2\mathbf{b}$.

(a) $\langle 7,-6\rangle$
(b) $\langle -4,17\rangle$
(c) $\langle 3,-1\rangle$
(d) $\langle -1,-18\rangle$
(e) $\langle 5,-6\rangle$

6. The curve given parametrically by the equations $x=\cos t$, $y=\sin^2 t$, $0\le t\le \pi/2$, forms

(a) a line segment.
(b) part of a parabola.
(c) part of a hyperbola.
(d) part of a circle.
(e) none of the above.

7. Refer to the graph of $y = f(x)$ given below, then decide which of the following statements is **false**.



(a) $\lim_{x\to 0} f(x) = 0$ (b) $\lim_{x\to 1^-} f(x) = 1$ (c) $\lim_{x\to 1^+} f(x) = 2$ (d) $\lim_{x\to 1} f(x) = 2$ (e) $\lim_{x\to 2^-} f(x) = 2$

8. Suppose f is a function defined on an open interval containing the point 3 and that $f(3) = 2$.
Which of the following statements is *always* true?

(a) $\lim_{x\to 3} f(x) = 2$.
(b) Either $\lim_{x\to 3^-} f(x)$ or $\lim_{x\to 3^+} f(x)$ must exist.
(c) If $\lim_{x\to 3} f(x)$ exists, then f is continuous at $x = 3$.
(d) If $\lim_{x\to 3} f(x) = 2$, then f is continuous at $x = 3$.
(e) If $\lim_{x\to 3} f(x) = 2$, then f is differentiable at $x = 3$.

9. Let f be the function defined by $f(x) = \begin{cases} x^2 + 3, & \text{if } x > 0; \\ 2x + 3, & \text{if } x < 0; \\ \dfrac{1}{x+1}, & \text{if } x = 0. \end{cases}$

Which of the following statements is true?

(a) f is not continuous at $x = 0$ because $\lim_{x\to 0} f(x)$ does not exist.
(b) f is not continuous at $x = 0$ because $\lim_{x\to 0} f(x)$ does not equal $f(0)$.
(c) f is not continuous at $x = -1$.
(d) f is continuous everywhere.
(e) All of the above statements are false.

10. One of the following intervals contains a number c such that $c^3 + 2c - 12 = \pi$. Which one?

(a) $(0, 1)$
(b) $(1, 2)$
(c) $(2, 3)$
(d) $(3, 4)$
(e) $(4, \infty)$

11. Evaluate $\lim_{x \to -\infty} \frac{\sqrt{x^2 + 4x}}{4x + 1}$.

(a) $-\frac{1}{4}$
(b) -1
(c) 0
(d) $\frac{1}{4}$
(e) $-\infty$

12. Find $\lim_{x \to 2^+} \frac{|3x - 6|}{6 - 3x}$.

(a) -1
(b) 0
(c) 1
(d) ± 1
(e) does not exist

13. Which of the following statements is true about the graph of $f(x) = \frac{(x^2 + 1)(x + 3)}{(4 - x^2)(x + 3)}$?

(a) The graph has one vertical asymptote $x = 2$ and no horizontal asymptotes.
(b) The graph has vertical asymptotes $x = 2$, $x = -2$, and horizontal asymptote $y = -1$.
(c) The graph has vertical asymptotes $x = 2$, $x = -2$, $x = -3$, and horizontal asymptote $y = -1$.
(d) $\lim_{x \to -3} f(x)$ does not exist.
(e) $\lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x)$.

Part 2: Work-Out Problems (48 points)

Partial credit is possible. *SHOW ALL STEPS!*

14. Differentiate; you need not simplify.

    (a) [3 points] $y = 2 - \frac{3}{x} + \frac{4}{x^2}$

    (b) [3 points] $f(x) = x + \sqrt[5]{2x^2}$

    (c) [3 points] $g(s) = \frac{4s - 7}{6s + 5}$

    (d) [3 points] $h(t) = \left(t^3 - 5t^2 + 6t + 7\right)\left(t^4 + t^3 + t^2 + t\right)$

15. A particle is moving in the xy-plane. Its position at time t is given by the position vector $\mathbf{r}(t) = \left(4t^2\right)\mathbf{i} + \left(t^3 - 9t - 2\right)\mathbf{j}$. Distances are in feet; times are in seconds.

    (a) [2 points] At what point is the particle at time $t = 3$? Write your answer in the form (a, b).

    (b) [3 points] What is the particle's velocity at time $t = 3$?

    (c) [2 points] What is the particle's speed at time $t = 3$? You need not simplify.

    (d) [2 points] At what time(s) will the particle's velocity be horizontal (parallel to $\mathbf{i}$)?

16. (a) [3 points] Find parametric equations for the line that contains the points $(2, 3)$ and $(4, -5)$.

(b) [2 points] The graph of $2x - 3y = 5$ is a line L. Which of the following statements is true? Circle the true statement.

i. The vector $2\mathbf{i} - 3\mathbf{j}$ is parallel to the line L.

ii. The vector $2\mathbf{i} - 3\mathbf{j}$ is perpendicular to the line L.

iii. The vector $2\mathbf{i} - 3\mathbf{j}$ has no special relationship with L.

(c) [2 points] Find a *unit* vector that is orthogonal to the vector $2\mathbf{i} + 5\mathbf{j}$.

17. Find the following limits.

(a) [3 points] $\lim_{x \to 2} \dfrac{x^2 - 4}{x^2 + x - 6}$

(b) [3 points] $\lim_{x \to \infty} \dfrac{1 + 2x - x^2}{1 - x + 2x^2}$

18. [5 points] Find an equation of the tangent line to the graph of $f(x) = \dfrac{3x-7}{x^2+5x-4}$ at the point where $x = 1$. Write your answer in the form $Ax + By = C$, where A, B, C are integers.

19. [4 points] Suppose f is a function such that $f(2) = f'(2) = 4$. Find $\lim_{x \to 2} \dfrac{(f(x))^2 - 16}{x-2}$. Your work needs to justify your answer. Do not skip steps.

20. [5 points] Let f be the function defined by $f(x) = \sqrt{x+3}$. Use the *definition of the derivative* to find $f'(1)$, the derivative of f at $x = 1$. Your work must clearly show that you know the definition of derivative.