Fall 2006 Math 151 Common Exam 2A Thu, 26/Oct/2006

Name (LAST, First): _____

Signature: _____

Instructor: _____

Section # _____

Seat # _____

For official use only!

QN	PTS	MAX
1–14		56
15		10
16		8
17		10
18		8
19		8
Total		100

Instructions

1. In **Part 1** (Problems 1–14), mark the correct choice on your ScanTron form using a No. 2 pencil. *For your own record, also mark your choices on your exam!* ScanTrons will be collected from all examinees **after 90 minutes** and will *not* be returned.

2. Be sure to write your **name**, **section** number, and **version** of the exam (**2A** or **2B**) on your ScanTron.

3. In **Part 2** (Problems 15–19), present your solutions in the space provided. **Show all your work** neatly and concisely, and **indicate your final answer clearly**. You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.

4. Neither calculators nor computers are permitted on this exam.

5. Please turn off all cell phones so as not to interrupt other students.

Part 1: Multiple Choice (56 points)

Read each question carefully. Each problem in Part 1 is worth **4 points**.

1. Evaluate $\lim_{x\to 0} \dfrac{3x\cos x}{\sin 2x}$.

   (a) $3/2$
   (b) 2
   (c) 3
   (d) π
   (e) ∞

2. Find the limit $\lim_{x\to 0} \cos\big(\sin(\sin x)\big)$.

   (a) π
   (b) 1
   (c) $\cos 1$
   (d) 0
   (e) does not exist

3. For what values of x is the tangent line to the graph of the function $f(x) = 2x - \sin 2x$ horizontal?

   (a) all integer multiples of π
   (b) all odd integer multiples of π
   (c) all integer multiples of $\frac{\pi}{2}$
   (d) all even integer multiples of π
   (e) there is no such x

4. Suppose that $x > 0$ and let s denote the distance between the points $(x, 0)$ and $(0, 1)$.
   If x is changing with time and $dx/dt = 2$, then $ds/dt =$?

   (a) $2x$
   (b) 2
   (c) $\dfrac{x}{x^2+1}$
   (d) $\dfrac{2x}{\sqrt{x^2+1}}$
   (e) $\dfrac{4x^2}{x^2+1}$

5. If $\log_3(3x+2) = 2$, what is x?

   (a) 0
   (b) $4/3$
   (c) $3/2$
   (d) $7/3$
   (e) does not exist

6. Let f and g be functions defined on $\mathbb{R}$ with the following properties: $f(3) = 1$, $f'(3) = 3$, $g(1) = 3$, $g'(1) = 2$. Let $h = f \circ g$ be the composition function defined by $h(x) = f(g(x))$. Compute $h'(1)$.

   (a) 1
   (b) 2
   (c) 3
   (d) 6
   (e) We do not have sufficient information to compute $h'(1)$.

7. Given that $f(x) = \sqrt{1-x}$, find the range of f^{-1}, the inverse of f.

   (a) $(-\infty, \infty)$
   (b) $[1, \infty)$
   (c) $(-\infty, 1]$
   (d) $[-1, 1]$
   (e) $[0, \infty)$

8. Find $f'(x)$ if $f(x) = x\cos^2\left(x^3\right)$.

   (a) $-6x^2\cos\left(x^3\right)\sin\left(x^3\right)$
   (b) $-3x^2\cos^2\left(x^3\right)\sin^2\left(x^3\right)$
   (c) $\cos^2\left(x^3\right) - 6x^3\cos\left(x^3\right)\sin\left(x^3\right)$
   (d) $6x^3\cos\left(x^3\right)$
   (e) $\cos^2\left(x^3\right) - 2x^4\cos\left(x^3\right)\sin\left(x^3\right)$

9. The position of a particle A is given by $\mathbf{r}(t) = (t+1)\,\mathbf{i} + \left(\sin\left(\frac{(t+1)\,\pi}{8}\right)\right)\mathbf{j}$.
Find the tangent vector to the path of the particle at time $t = 3$.

(a) $\mathbf{i}$
(b) $\mathbf{j}$
(c) $4\mathbf{i} - \mathbf{j}$
(d) $\mathbf{i} + \frac{1}{8}\pi\,\mathbf{j}$
(e) $4\mathbf{i} + \mathbf{j}$

10. Let p be a polynomial of degree 2 such that $p(1) = 1$, $p'(0) = 2$ and $p''(1) = 2$. What is the value of $p(-1)$?

(a) -3
(b) -1
(c) 0
(d) 1
(e) 3

11. The parametric curve $x = t\left(t^2 - 1\right)$, $y = t^2 - 1$, crosses itself at $(0, 0)$.
Find the angle between the two tangent lines at $(0, 0)$.

(a) 0°
(b) 30°
(c) 45°
(d) 60°
(e) 90°

12. Let $L(x)$ be the linear approximation of $f(x) = \sqrt{x}$ at $a = 100$. Compute $L(90)$.

    (a) 9.3
    (b) 9.4
    (c) 9.5
    (d) 9.6
    (e) 9.7

13. Let $q(x)$ be the quadratic approximation of $p(x) = x^2 - 2x + 1$ about $a = 2$. Compute $q(1)$.

    (a) -1
    (b) 0
    (c) 1
    (d) 2
    (e) 4

14. Suppose g is the inverse function of f. Given that $f(1) = 2$, $f'(1) = 3$, and $f'(2) = 3$, we can deduce that $g'(2)$ equals

    (a) 3
    (b) 2
    (c) 1
    (d) $1/2$
    (e) $1/3$

Part 2: Work-Out Problems (44 points)

Show all your work neatly and concisely, and **indicate your final answer clearly**. You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it. Partial credit is possible.

15. Consider the curve C given by the parametric equations

$$x = te^{-t}, \qquad y = (2t+1)^{1/3}, \qquad -\infty < t < \infty.$$

(a) [3 points] Find dx/dt and dy/dt.

(b) [3 points] Find dy/dx in terms of the parameter t.

(c) [4 points] Obtain an equation of the tangent line to C at the point $(0, 1)$.

16. [8 points] Find an equation of the tangent line to the curve $x^2 - xy + y^3 = 25$ at $(1, 3)$.

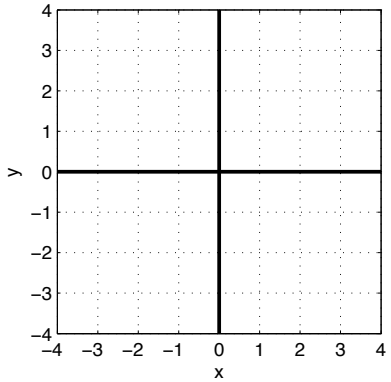
17. The position of a particle at time t is given by $\mathbf{r}(t) = (2\cos 2t)\,\mathbf{i} + (2\sin 2t)\,\mathbf{j}$.

(a) [3 points] Find the velocity of the particle at time $t = \pi/12$.

(b) [2 points] Find the speed of the particle at a general time t. Simplify your answer.

(c) [3 points] Show that at any time t, the particle's velocity and acceleration are orthogonal (perpendicular).

(d) [2 points] Sketch the path of the particle on the grid below and indicate its direction of motion.



18. Let $f(x) = \dfrac{x}{\sqrt{7-3x}}$.

(a) [2 points] Find $f'(2)$.

(b) [2 points] Compute $f'(f(2))$.

(c) [2 points] Let $h = f \circ f$ be the composite function defined by $h(x) = f(f(x))$. Use the Chain Rule to compute $h'(2)$.

(d) [2 points] Now write the linear approximation $L(x)$ of $h(x)$ at $a = 2$.

$L(x) =$ _____

Please turn the page over for the LAST problem!

19. [8 points] The following rates of change refer to a triangle.

- The height of the triangle is increasing at a rate of 2 cm/min.
- The area of the triangle is decreasing at a rate of 4 cm^2/min.
- At what rate is the base of the triangle changing when the height of the triangle is 10 cm and the area of the triangle is 100 cm^2?

Clearly define all variables used in your solution. Include units with your answer.