Fall 2006 Math 151 Common Exam 3B Tue, 28/Nov/2006

Name (LAST, First): ______________________________

Signature: ______________________________

Instructor: ______________________________

Section # ______________________________

Seat # ______________________________

For official use only!

QN	PTS	MAX
1–13		52
14		8
15		9
16		3
17		5
18		6
19		10
20		7
Total		100

Instructions

1. Time allowed: *2 hours*. Be sure to write your **name**, **section** number, and **version** of the exam (**3A** or **3B**) on your ScanTron.
2. In **Part 1** (Problems 1–13), mark the correct choice on your ScanTron form using a No. 2 pencil. *For your own record, also mark your choices on your exam!* ScanTrons will be collected from all examinees **after 90 minutes** and will *not* be returned.
3. In **Part 2** (Problems 14–20), present your solutions in the space provided. **Show all your work** neatly and concisely, and **indicate your final answer clearly**. You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.
4. Neither calculators nor computers are permitted on this exam.
5. Please turn off all cell phones so as not to interrupt other students.
6. Remember: "An Aggie does not lie, cheat, or steal or tolerate those who do."

Part 1: Multiple Choice (52 points)

Read each question carefully. Each problem in Part 1 is worth **4 points**.

1. Compute the derivative $\dfrac{d}{dx}\left(\tan^{-1}(\sin x)\right)$.

   (a) $\dfrac{\cos x}{x^2+1}$
   (b) $\dfrac{1}{1+\sin^2 x}$
   (c) $\sec^2(\cos x)$
   (d) $\sec x$
   (e) $\dfrac{\cos x}{1+\sin^2 x}$

2. Some leftover Thanksgiving turkey is placed into a refrigerator at 0 °C. The rate of cooling of the turkey is equal to half its temperature. After ln (9) hours, the turkey is taken out. What will be its temperature, if its temperature initially was 57 °C?

   (a) 0 °C
   (b) 28.5 °C
   (c) 48 °C
   (d) 19 °C
   (e) 15 °C

3. One of the following is an antiderivative for the function $f(x) = x^3 + 6x^2$. Which one is it?

   (a) $F(x) = \frac{1}{3}x^4 + 3x^3$
   (b) $F(x) = x^4 + 6x^3$
   (c) $F(x) = \frac{1}{4}x^4 + 2x^3 + 9$
   (d) $F(x) = 3x^4 + 12x^3 + C$, where C is any constant
   (e) $F(x) = \frac{1}{3}x^2 + 3x + 1$

4. Evaluate $\lim_{x\to 0} (\cos x)^{x^{-2}}$.

(a) $e^{-1/2}$
(b) 0
(c) e
(d) 1
(e) The limit does not exist.

5. Write the sum $\frac{1}{\sqrt{3}} + \frac{1}{\sqrt{4}} + \frac{1}{\sqrt{5}} + \cdots + \frac{1}{\sqrt{2006}}$ using summation notation.

(a) $\sum_{n=1}^{2004} \frac{1}{\sqrt{n}}$
(b) $\sum_{n=3}^{2006} \frac{1}{\sqrt{n+2}}$
(c) $\sum_{n=1}^{2006} \frac{1}{\sqrt{n+2}}$
(d) $\sum_{n=1}^{2006} \frac{1}{\sqrt{n}}$
(e) $\sum_{n=3}^{2006} \frac{1}{\sqrt{n}}$

6. Calculate $\cos\left(\tan^{-1} 2\right)$.

(a) $1/5$
(b) $1/\sqrt{5}$
(c) $-1/\sqrt{5}$
(d) $\sqrt{5}$
(e) It does not exist.

7. What is the largest interval on which $y = e^{-x^2/2}$ is concave down?

(a) $-\infty < x < \infty$
(b) $0 < x < \infty$
(c) $-1 < x < 1$
(d) $0 < x < 1$
(e) $-1 < x < 0$

8. Calculate the limit $\lim_{x \to 2} \dfrac{x^3 - 12x + 16}{x^3 - 3x^2 + 4}$.

(a) 6
(b) 2
(c) The limit does not exist.
(d) 1
(e) 0

9. Let f and g be functions defined on the integers. Suppose we know that $\sum_{n=1}^{10} f(n) = 101$ and $\sum_{n=1}^{10} g(n) = 35$.

Calculate $\sum_{n=1}^{10} (5f(n) + 2g(n))$.

(a) 635
(b) 500
(c) More information is needed.
(d) 136
(e) 575

10. Find the *absolute maximum* value of $f(x) = \dfrac{1}{x^2 - 6x + 10}$ on the interval $[0, 4]$.

(a) 2
(b) 1
(c) $1/10$
(d) $1/\sqrt{10}$
(e) $1/2$

11. Differentiate $y = (1+x)^x$.

(a) $dy/dx = (1+x)^x \ln(1+x)$

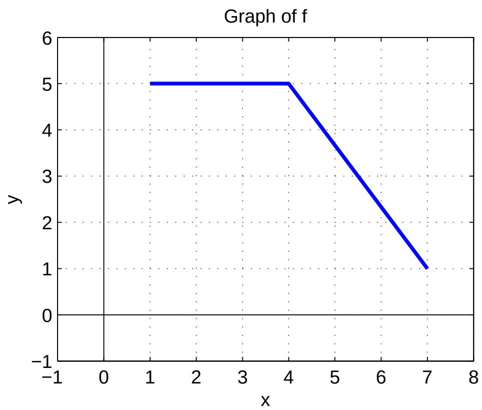
(b) $\dfrac{dy}{dx} = \dfrac{x}{1+x}$

(c) $dy/dx = x^x$

(d) $dy/dx = x\,(1+x)^{x-1}$

(e) $dy/dx = (1+x)^x \ln(1+x) + x\,(1+x)^{x-1}$

12. Below is the graph of a function f. Find $\int_1^7 f(x)\,dx$.



(a) 23.5

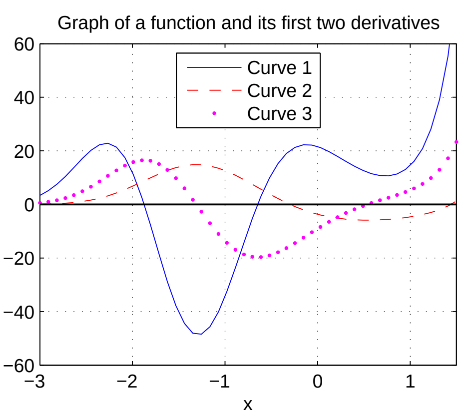
(b) 24.0

(c) 26.0

(d) 25.0

(e) 24.5

13. Here is a plot of the graphs of a function f and its first two derivatives. Which is which?



(a) Curve 1 is f; curve 2 is f'; curve 3 is f''.

(b) Curve 1 is f; curve 2 is f''; curve 3 is f'.

(c) Curve 1 is f'; curve 2 is f; curve 3 is f''.

(d) Curve 1 is f'; curve 2 is f''; curve 3 is f.

(e) Curve 1 is f''; curve 2 is f; curve 3 is f'

Part 2: Work-Out Problems (48 points)

Show all your work neatly and concisely, and **indicate your final answer clearly**. You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it. Partial credit is possible.

14. [8 points] Calculate the limit $\lim_{x \to 0} \left(\frac{1}{x} - \frac{1}{\ln(1+x)} \right)$.

15. Differentiate the following functions.

(a) [3 points] $y = \ln\left(\frac{(1+x)^8\sqrt{x}}{\sqrt[6]{1+2x}}\right)$ [HINT: Use logarithm rules to simplify first. . .]

(b) [3 points] $f(x) = e^{3x} + (3x)^e + e^{\pi} + (3+e)^x$

(c) [3 points] $g(x) = \ln|x| + \ln\left(\ln\left(x^3\right)\right) + \log_5\left(x^2+4\right)$

16. [3 points] State the Mean Value Theorem.

17. [5 points] A particle moves in the xy-plane. Its acceleration is $\mathbf{a}(t) = \mathbf{i} + t\mathbf{j}$. If the particle's velocity at time $t = 0$ is $\mathbf{v}(0) = \mathbf{i} - \mathbf{j}$ and its position at time $t = 1$ is $\mathbf{r}(1) = 3\mathbf{i} + \mathbf{j}$, find its position $\mathbf{r}(t)$ at time t in general.

18. [6 points; 1 point each] Let $f(x) = \cos^{-1} x$. Answer these six items.

(a) The domain of f is: ____________________________

(b) The range of f is: ____________________________

(c) $\cos^{-1}\left(\cos \frac{3}{2}\pi\right) =$ ____________________________

(d) $f'(0) =$ ____________________________

(e) $\sin\left(\cos^{-1} \frac{1}{4}\right) =$ ____________________________

(f) $\cos\left(\cos^{-1} \frac{1}{4}\right) =$ ____________________________

19. Consider the function $f(x) = x^3 - 3x^2 - 45x - 22$.

(a) [2 points] Find the critical numbers of f.

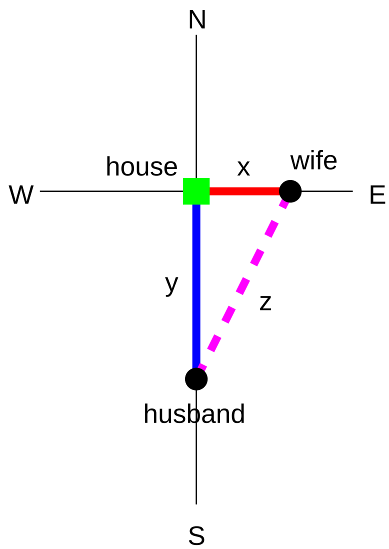
(b) [4 points] For each critical number c, determine *with justification* whether $(c, f(c))$ is a local maximum, a local minimum, or neither.

(c) [2 points] Find the x-coordinate of the point of inflection of f.

(d) [2 points] Find the absolute maximum of f on $[-1, 0]$ and where it occurs. Justify your assertions.

Please turn the page over for the LAST problem!

20. A man sets off to go home from a point 10 miles south of his house. He walks due north at a speed of 4 miles per hour. At the same moment his wife sets off from the house, heading due east at a speed of 3 miles per hour. **In the following, clearly define all variables used in your solution. Include units with your answers.** Here is a diagram to help get you started.



(a) [5 points] At what time after setting out are they closest together?

(b) [2 points] What is the distance between them when they are closest together?