

Math 151 Fall 2008 Exam I Solutions-Form A

1. C: First, we will find the slope of the given line. $2x + 4y = 11$ is equivalent to $y = -\frac{1}{2}x + \frac{11}{4}$. Hence the slope of the line is $-\frac{1}{2}$. The only vector that has this direction is $\langle -2, 1 \rangle$.

2. B:
$$\lim_{x \rightarrow 4} \frac{x-4}{2-\sqrt{x}} = \lim_{x \rightarrow 4} \frac{(\sqrt{x}-2)(\sqrt{x}+2)}{2-\sqrt{x}}$$

$$= \lim_{x \rightarrow 4} (-1)(\sqrt{x}+2) = -4.$$

3. E: Use the quotient rule to differentiate $f(x)$. $f'(x) = \frac{(-1)(1+x) - (1-x)}{(1+x)^2} = \frac{-2}{(1+x)^2}$. Hence the slope of the tangent line is $m = f'(-2) = -2$. Also, when $x = -2$, $y = -3$. Thus the equation of the tangent line is $y - (-3) = -2(x + 2)$ or $y = -2x - 7$.

4. A: $W = |\mathbf{F}||\mathbf{D}|\cos(\theta)$. Here, $|\mathbf{F}| = 2$ N, $|\mathbf{D}| = 6$ m and $\theta = 60^\circ$. Hence $W = (2\text{N})(6\text{m})\cos(60^\circ) = 6$ N-m.

5. E: Since $x < 3$, $|x - 3| = -(x - 3)$. Thus

$$\lim_{x \rightarrow 3^-} \frac{|x-3|}{x^2-9} = \lim_{x \rightarrow 3^-} \frac{-(x-3)}{(x+3)(x-3)}$$

$$= \lim_{x \rightarrow 3^-} \frac{-1}{x+3} = -\frac{1}{6}$$

6. B: $\lim_{x \rightarrow 5^-} \frac{x-7}{x(x-5)} = \infty$ since $x = 5$ is a vertical asymptote and $\frac{x-7}{x(x-5)} > 0$ for $x < 5$.

7. E: First, note $f(x) = x^3 + 2x^2 - 42$ is continuous, so we will apply the Intermediate Value Theorem. Since $f(2) = -26 < 0$ and $f(3) = 3 > 0$, there is a solution to $f(c) = 0$ on the interval $(2, 3)$.

8. D: Recall $f'(1.5)$ is the slope of the tangent to the graph of $f(x)$ at $x = 1.5$. Since $f(x)$ is linear on the interval $[1, 2]$, $f'(1.5)$ is equivalent to the slope of the line segment from $x = 1$ to $x = 2$. Hence $f'(1.5) = \frac{5-3}{1} = 2$.

9. D: First we will choose two points on the line. Let $A(0, 1)$ and $B(1, 3)$. If we then define

$\mathbf{a} = \vec{AB} = \langle 1, 2 \rangle$ and $\mathbf{b} = \vec{AP} = \langle 1, 4 \rangle$, then

$$d = |\text{comp}_{\mathbf{a}} \mathbf{b}| = \left| \frac{\langle -2, 1 \rangle \cdot \langle 1, 4 \rangle}{|\langle -2, 1 \rangle|} \right| = \frac{2}{\sqrt{5}}.$$

10. (i) First note that both $x + 8a$ and ax^2 are continuous everywhere on their domains. Thus all we need to do is find the value of a so that

$$\lim_{x \rightarrow 3^+} f(x) = \lim_{x \rightarrow 3^-} f(x) = f(3). \text{ Now,}$$

$$\lim_{x \rightarrow 3^+} f(x) = \lim_{x \rightarrow 3^+} ax^2 = 9a = f(3) \text{ and}$$

$$\lim_{x \rightarrow 3^-} f(x) = \lim_{x \rightarrow 3^-} (x + 8a) = 3 + 8a. \text{ Thus}$$

$$9a = 3 + 8a, \text{ yielding } a = 3.$$

(ii) Substituting $a = 3$ into $f(x)$ yields

$$f(x) = \begin{cases} x + 24 & \text{if } x < 3 \\ 3x^2 & \text{if } x \geq 3 \end{cases}$$

Now, $\lim_{x \rightarrow 3^+} f(x) = \lim_{x \rightarrow 3^+} (3x^2) = 27$ and $\lim_{x \rightarrow 3^-} f(x) =$

$$\lim_{x \rightarrow 3^-} (x + 24) = 27. \text{ Thus}$$

$$\lim_{x \rightarrow 3} f(x) = 27.$$

11. Multiply by the conjugate:

$$\lim_{x \rightarrow \infty} (\sqrt{x^2 + 2x} - x) \frac{\sqrt{x^2 + 2x} + x}{\sqrt{x^2 + 2x} + x}$$

$$= \lim_{x \rightarrow \infty} \frac{2x}{\sqrt{x^2 + 2x} + x}$$

$$= \lim_{x \rightarrow \infty} \frac{2x}{|x| + x}$$

$$= \lim_{x \rightarrow \infty} \frac{2x}{2x} = 1$$

12. (i) By the quotient rule,

$$h'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}. \text{ Thus}$$

$$h'(4) = \frac{f'(4)g(4) - f(4)g'(4)}{(g(4))^2} = -\frac{5}{2}$$

(ii) By the product rule,

$$w'(x) = f'(x)g(x) + f(x)g'(x), \text{ thus}$$

$$w'(4) = f'(4)g(4) + f(4)g'(4) = 14$$

13. (i) $\text{proj}_{\mathbf{a}} \mathbf{b} = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}|^2} \mathbf{a}$

$$= \frac{\langle 2, 3 \rangle \cdot \langle 1, 4 \rangle}{|\langle 2, 3 \rangle|^2} \langle 2, 3 \rangle$$

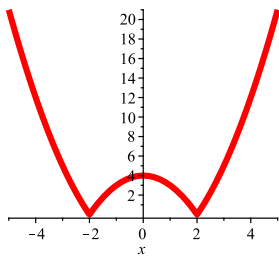
$$= \frac{14}{13} \langle 2, 3 \rangle$$

$$= \left\langle \frac{28}{13}, \frac{42}{13} \right\rangle$$

(ii) $\text{comp}_{\mathbf{a}} \mathbf{b} = \frac{\langle 2, 3 \rangle \cdot \langle 1, 4 \rangle}{|\langle 2, 3 \rangle|} = \frac{14}{\sqrt{13}}$

$$\begin{aligned}
14. \quad f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\
&= \lim_{h \rightarrow 0} \frac{\frac{1}{x+h+1} - \frac{1}{x+1}}{h} \\
&= \lim_{h \rightarrow 0} \frac{\frac{1}{x+h+1} - \frac{1}{x+1}}{h} \frac{(x+h+1)(x+1)}{(x+h+1)(x+1)} \\
&= \lim_{h \rightarrow 0} \frac{x+1 - (x+h+1)}{h(x+h+1)(x+1)} \\
&= \lim_{h \rightarrow 0} \frac{-h}{h(x+h+1)(x+1)} \\
&= \frac{-1}{(x+1)^2}
\end{aligned}$$

15. (i)



(ii) $f(x)$ is not differentiable at $x = \pm 2$.

$$\text{(iii) } f'(x) = \begin{cases} -2x & \text{if } -2 < x < 2 \\ 2x & \text{if } |x| > 2 \end{cases}$$