Math 151 Fall 2008 Exam I Solutions-Form B

1. C: Recall $f'(1.5)$ is the slope of the tangent to the graph of $f(x)$ at $x = 1.5$. Since $f(x)$ is linear on the interval $[1, 2]$, $f'(1.5)$ is equivalent to the slope of the line segment from $x = 1$ to $x = 2$. Hence $f'(1.5) = \frac{5-3}{1} = 2$.

2. B: Use the quotient rule to differentate $f(x)$. $f'(x) = \dfrac{(-1)(1+x) - (1-x)}{(1+x)^2} = \dfrac{-2}{(1+x)^2}$. Hence the slope of the tangent line is $m = f'(-2) = -2$. Also, when $x = -2$, $y = -3$. Thus the equation of the tangent line is $y - (-3) = -2(x + 2)$ or $y = -2x - 7$.

3. E: First, we will find the slope of the given line. $2x + 4y = 11$ is equivalent to $y = -\frac{1}{2}x + \frac{11}{4}$. Hence the slope of the line is $-\frac{1}{2}$. The only vector that has this direction is $\langle -2, 1\rangle$.

4. A: First, note $f(x) = x^3 + 2x^2 - 42$ is continuous, so we will apply the Intermediate Value Theorem. Since $f(2) = -26 < 0$ and $f(3) = 3 > 0$, there is a solution to $f(c) = 0$ on the interval $(2, 3)$.

5. A: $\displaystyle\lim_{x\to 4} \frac{x-4}{2-\sqrt{x}} = \lim_{x\to 4} \frac{(\sqrt{x}-2)(\sqrt{x}+2)}{2-\sqrt{x}}$
$= \displaystyle\lim_{x\to 4} (-1)(\sqrt{x}+2) = -4$.

6. D: $W = |\mathbf{F}||\mathbf{D}|\cos(\theta)$. Here, $|\mathbf{F}| = 2$ N, $|\mathbf{D}| = 6$ m and $\theta = 60°$. Hence $W = (2\text{N})(6\text{m})\cos(60°)$
$= 6$ N-m.

7. C: Since $x < 3$, $|x - 3| = -(x - 3)$. Thus
$\displaystyle\lim_{x\to 3^-} \frac{|x-3|}{x^2-9} = \lim_{x\to 3^-} \frac{-(x-3)}{(x+3)(x-3)}$
$= \displaystyle\lim_{x\to 3^-} \frac{-1}{x+3} = -\frac{1}{6}$

8. C: $\displaystyle\lim_{x\to 5^-} \frac{x-7}{x(x-5)} = \infty$ since $x = 5$ is a vertical asymptote and $\frac{x-7}{x(x-5)} > 0$ for $x < 5$.

9. A: First we will choose two points on the line. Let $A(0, 1)$ and $B(1, 3)$. If we then define
$\mathbf{a} = \vec{AB} = \langle 1, 2\rangle$ and $\mathbf{b} = \vec{AP} = \langle 1, 4\rangle$, then
$d = |\text{comp}_{\mathbf{a}^\perp}\mathbf{b}| = \left|\dfrac{\langle -2,1\rangle \cdot \langle 1,4\rangle}{|\langle -2,1\rangle|}\right| = \dfrac{2}{\sqrt{5}}$.

10. (i) First note that both $x + 2a$ and ax^2 are continuous everywhere on their domains. Thus all we need to do is find the value of a so that
$\displaystyle\lim_{x\to 2^+} f(x) = \lim_{x\to 2^-} f(x) = f(2)$. Now,
$\displaystyle\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} ax^2 = 4a = f(2)$ and
$\displaystyle\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} (x + 2a) = 2 + 2a$. Thus
$4a = 2 + 2a$, yielding $a = 1$.
(ii) Substituting $a = 1$ into $f(x)$ yields
$$f(x) = \begin{cases} x+2 & \text{if } x < 3 \\ x^2 & \text{if } x \geq 3 \end{cases}$$
Now, $\displaystyle\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} (x^2) = 4$ and
$\displaystyle\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} (x + 2) = 4$. Thus
$\displaystyle\lim_{x\to 2} f(x) = 4$.

11. Multiply by the conjugate:
$$\lim_{x\to\infty} (\sqrt{x^2+2x} - x)\frac{\sqrt{x^2+2x}+x}{\sqrt{x^2+2x}+x}$$
$$= \lim_{x\to\infty} \frac{2x}{\sqrt{x^2+2x}+x}$$
$$= \lim_{x\to\infty} \frac{2x}{|x|+x}$$
$$= \lim_{x\to\infty} \frac{2x}{2x} = 1$$

12. (i) By the quotient rule,
$h'(x) = \dfrac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$. Thus
$h'(4) = \dfrac{f'(4)g(4) - f(4)g'(4)}{(g(4))^2} = -\dfrac{10}{9}$
(ii) By the product rule,
$w'(x) = f'(x)g(x) + f(x)g'(x)$, thus
$w'(4) = f'(4)g(4) + f(4)g'(4) = -14$

13. (i) $\text{proj}_{\mathbf{a}}\mathbf{b} = \dfrac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}|^2}\mathbf{a}$
$$= \frac{\langle 4,5\rangle \cdot \langle 1,3\rangle}{|\langle 4,5\rangle|^2}\langle 4,5\rangle$$
$$= \frac{19}{41}\langle 4,5\rangle$$
$$= \left\langle \frac{76}{41}, \frac{95}{41}\right\rangle$$
(ii) $\text{comp}_{\mathbf{a}}\mathbf{b} = \dfrac{\langle 4,5\rangle \cdot \langle 1,3\rangle}{|\langle 4,5\rangle|} = \dfrac{19}{\sqrt{41}}$

14. $$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\dfrac{1}{x+h+2} - \dfrac{1}{x+2}}{h}$$

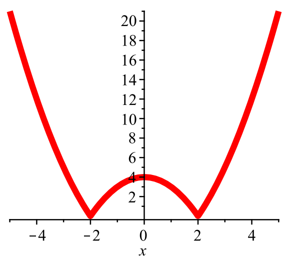
$$= \lim_{h\to 0} \frac{\dfrac{1}{x+h+2} - \dfrac{1}{x+2}}{h} \frac{(x+h+2)(x+2)}{(x+h+2)(x+2)}$$

$$= \lim_{h\to 0} \frac{x+2-(x+h+2)}{h(x+h+2)(x+2)}$$

$$= \lim_{h\to 0} \frac{-h}{h(x+h+2)(x+2)}$$

$$= \frac{-1}{(x+2)^2}$$

15. (i)



(ii) $f(x)$ is not differentiable at $x = \pm 2$.

(iii) $$f'(x) = \begin{cases} -2x & \text{if } -2 < x < 2 \\ 2x & \text{if } |x| > 2 \end{cases}$$