Math 151 Fall 2008 Exam II Solutions-Form A

1. C: Let $f(x) = x^5 - 2x + 5$. Newton's Method says if x_1 is the first guess to $f(x) = 0$, then the second approximation is given by $x_2 = x_1 - \dfrac{f(x_1)}{f'(x_1)}$. Thus if $x_1 = 1$, $x_2 = 1 - \dfrac{f(1)}{f'(1)} = 1 - 4/3 = -\dfrac{1}{3}$.

2. A: $\displaystyle\lim_{\theta\to 0} \frac{\sin^2(3\theta)}{\theta^2} = \lim_{\theta\to 0}\left(\frac{\sin(3\theta)}{\theta}\right)\left(\frac{\sin(3\theta)}{\theta}\right)$
$\displaystyle= 9\lim_{\theta\to 0}\left(\frac{\sin(3\theta)}{3\theta}\right)\left(\frac{\sin(3\theta)}{3\theta}\right) = 9(1)(1) = 9$

3. D: First we will find the tangent vector at $t = 0$ and then make it a unit vector by dividing by the magnitude:
$\mathbf{r}(t) = \langle e^{2t}, t\cos t\rangle$ thus
$\mathbf{r}'(t) = \langle 2e^{2t}, \cos t - t\sin t\rangle$. Therefore
$\mathbf{r}'(0) = \langle 2e^0, \cos(0) - (0)\sin(0)\rangle = \langle 2, 1\rangle$. The unit vector is $\dfrac{\langle 2,1\rangle}{|\langle 2,1\rangle|} = \left\langle \dfrac{2}{\sqrt{5}}, \dfrac{1}{\sqrt{5}}\right\rangle$.

4. C: To solve $\ln(x+e) + \ln(x-e) = 2 + \ln(3)$, we will use logarithm properities:
$\ln(x+e) + \ln(x-e) - \ln(3) = 2$
$\to \ln\left(\dfrac{(x+e)(x-e)}{3}\right) = 2$
$\to \dfrac{(x+e)(x-e)}{3} = e^2$
$\to (x+e)(x-e) = 3e^2$
$\to x^2 - e^2 = 3e^2$, yielding $x = \pm 2e$. Now, the domain of $\ln(x+e) + \ln(x-e) = 2 + \ln(3)$ is $x > e$, hence $x = -2e$ is extraneous, so the only solution is $x = 2e$.

5. A: Recall if g is the inverse of f, then
$g'(a) = \dfrac{1}{f'(g(a))}$. Thus $g'(2) = \dfrac{1}{f'(g(2))}$.
Since $f(3) = 2$, $g(2) = 3$. $g'(2) = \dfrac{1}{f'(3)} = \dfrac{1}{7}$

6. B: By the chain rule, if $h(x) = f(g(x))$,
$h'(x) = f'(g(x))g'(x)$.
Thus $h'(-3) = f'(g(-3))g'(-3) = f'(-2)(4) = 44$

7. C: $f(t) = 2\sin t - 3\cos t$,
$v(t) = f'(t) = 2\cos t + 3\sin t$ and
$a(t) = v'(t) = -2\sin t + 3\cos t$. Evaluating these at $t = \dfrac{\pi}{6}$ yields $v\left(\dfrac{\pi}{6}\right) = \sqrt{3} + \dfrac{3}{2}$ and
$a\left(\dfrac{\pi}{6}\right) = -1 + \dfrac{3\sqrt{3}}{2}$

8. D: The quadratic approximation for $f(x)$ at $x = 1$ is $Q(x) = f(1) + f'(1)(x-1) + \dfrac{f''(1)}{2}(x-1)^2$.
$f(1) = 2$, $f'(1) = -2$ and $f''(1) = 4$. Thus
$Q(x) = 2 - 2(x-1) + 2(x-1)^2$, and
$Q(\frac{1}{2}) = 2 - 2(\frac{1}{2} - 1) + 2(\frac{1}{2} - 1)^2 = \dfrac{7}{2}$.

9. B: $\displaystyle\lim_{x\to 0^-} e^{1/x} = 0$ since $\displaystyle\lim_{x\to 0^-} 1/x = -\infty$
and $\displaystyle\lim_{x\to 0^-} e^{1/x} = e^{\left(\lim_{x\to 0^-} 1/x\right)} = \lim_{y\to -\infty} e^y = 0$

10. A: Let $y = \dfrac{1-x}{4x+3}$. Interchange x and y:
$x = \dfrac{1-y}{4y+3}$. Solve for y: $x(4y+3) = 1 - y$
$4xy + y = 1 - 3x$, hence $y = \dfrac{1-3x}{4x+1}$.

11. (i) $f(x) = \tan^3 x + \tan(x^3)$. By the chain rule,
$f'(x) = 3\tan^2 x\sec^2 x + \sec^2(x^3)(3x^2)$.
(ii) $g(t) = \sqrt{1+\sqrt{t}} = (1 + t^{1/2})^{1/2}$. By the chain rule,
$g'(t) = \dfrac{1}{2}(1 + t^{1/2})^{-1/2}\dfrac{1}{2}t^{-1/2}$
$= \dfrac{1}{4\sqrt{t}\sqrt{1+\sqrt{t}}}$

12. We are given $\dfrac{dV}{dt} = \dfrac{3}{2}$. We want to find $\dfrac{dh}{dt}$ when $h = 4$. The volume of a cone is $V = \dfrac{1}{3}\pi r^2 h$. To get V in terms of h only, we will use similar triangles. $\dfrac{r}{h} = \dfrac{2}{6}$, thus $r = \dfrac{1}{3}h$. Substitute this in for r:
$V = \dfrac{1}{3}\pi(\dfrac{1}{3}h)^2 h = \dfrac{\pi}{27}h^3$. Differentiate with respect to time: $\dfrac{dV}{dt} = \dfrac{\pi}{9}h^2\dfrac{dh}{dt}$. Now substitute $\dfrac{dV}{dt} = \dfrac{3}{2}$ and $h = 4$ yields $\dfrac{3}{2} = \dfrac{\pi}{9}(4)^2\dfrac{dh}{dt}$, thus $\dfrac{dh}{dt} = \dfrac{27}{32\pi}$ inches per second.

13. First we will differentiate implicitly with respect to x:

$y^2 \sin 2x = 8 - 2y$. Using the product rule and chain rule,

$2y\frac{dy}{dx}\sin 2x + y^2 2\cos 2x = -2\frac{dy}{dx}$. Solve for $\frac{dy}{dx}$.

$\frac{dy}{dx}(2y\sin 2x + 2) = -2y^2\cos 2x$, thus

$\frac{dy}{dx} = \frac{-2y^2\cos 2x}{2y\sin 2x + 2}$. To find the slope of the tangent line, we will substitute the point $\left(\frac{\pi}{4}, 2\right)$ into $\frac{dy}{dx}$.

$m = \frac{-2(2)^2\cos\frac{\pi}{2}}{2(2)\sin(\frac{\pi}{2}) + 2} = 0$. Thus the equation of the line is $y - 2 = 0(x - \pi/4)$, hence $y = 2$.

14. (i) To find the tangent line, we need a point and the slope. To find the point, substitute $t = 1$ into the parametric equations:

$x = t^2 - 10t$, $y = t^3 - 3t^2$, so when $t = 1$, $x = -9$ and $y = -2$. Thus the point is $(-9, -2)$. Now, the slope of the tangent line is

$m = \frac{dy/dt}{dx/dt}$ evaluated at $t = 1$. $\frac{dy}{dx} = \frac{3t^2 - 6t}{2t - 10}$, thus when $t = 1$, $m = \frac{3}{8}$. So the equation of the tangent line is $y + 2 = \frac{3}{8}(x + 9)$.

(ii) The tangent line is vertical where $\frac{dx}{dt} = 0$ and $\frac{dy}{dt} \neq 0$. This happens when $2t - 10 = 0$, thus $t = 5$. Substitute this into $x = t^2 - 10t$, $y = t^3 - 3t^2$, we get the point $(-25, 50)$

(iii) The tangent line is horizontal where $\frac{dy}{dt} = 0$ and $\frac{dx}{dt} \neq 0$. This happens when $3t^2 - 6t = 0$, thus $t = 0$ and $t = 2$. Substitute this into $x = t^2 - 10t$, $y = t^3 - 3t^2$, we get the points $(0, 0)$ and $(-16, -4)$.

15. Using differentials, $f(a + dx) \approx f(a) + f'(a)dx$, where $f(x) = \sqrt{x}$, $a = 16$ and $dx = 0.03$. Thus

$f(16.03) = \sqrt{16.03} \approx f(16) + f'(16)(0.03)$

$= 4 + \frac{1}{8}(0.03) = 4.00375$

Using Linear Approximation: Find the linear approximation for $f(x) = \sqrt{x}$ at $x = 16$.

$L(x) = f(16) + f'(16)(x - 16)$.

Now, $f(16) = \sqrt{16} = 4$ and $f'(16) = \frac{1}{2\sqrt{16}} = \frac{1}{8}$.

Hence $L(x) = 4 + \frac{1}{8}(x - 16)$. Now, to approximate $\sqrt{16.03}$, substitute $x = 16.3$ in $L(x)$ for x:

$L(16.3) = 4 + \frac{1}{8}(16.3 - 16) = 4 + \frac{1}{8}(0.03) = 4.00375$

16. To find where the tangent line is horizontal, we need to solve $f'(x) = 0$ for $0 \leq x \leq 2\pi$. $f(x) = x + 2\sin x$, thus $f'(x) = 1 + 2\cos x$. Solve $f'(x) = 0$ gives $\cos x = -\frac{1}{2}$. For $0 \leq x \leq 2\pi$, this happens when $x = \frac{2\pi}{3}$ and $x = \frac{4\pi}{3}$.