Math 151 Fall 2008 Exam III Solutions-Form A

1. D: $\lim_{x\to 0}\frac{2^x-5^x}{x}$ is of the form $\frac{0}{0}$, so we will apply L'Hospital's rule.
$\lim_{x\to 0}\frac{2^x-5^x}{x}=\lim_{x\to 0}\frac{2^x\ln 2-5^x\ln 5}{1}=\ln 2-\ln 5$
$=\ln\frac{2}{5}$

2. D: $y=y_0e^{kt}$. Since the population triples every 5 minutes, $y(5)=3y_0$. Thus $3y_0=y_0e^{5k}$. Solve for k: $3=e^{5k}$, thus $\ln 3=\ln e^{5k}$, yielding $\frac{\ln 3}{5}=k$.

3. B: $f(x)=\ln(x+\ln x)$. By the chain rule,
$f'(x)=\frac{1+1/x}{x+\ln x}$. Thus $f'(e)=\frac{1+1/e}{e+\ln e}=\frac{1}{e}$.

4. C: $f'(x)=3\cos x+5\sin x$. Antidifferentiating,
$f(x)=3\sin x-5\cos x+C$. Now, we are given that $f(0)=4$, so $4=3\sin(0)-5\cos(0)+C$, hence
$C=9$. Thus $f(x)=3\sin x-5\cos x+9$, and therefore $f(\pi)=14$.

5. E: Note that $y=\sqrt{9-x^2}$ for $0\le x\le 3$ is a quarter of the circle $x^2+y^2=9$ in the first quadrant. Thus $\int_0^3\sqrt{9-x^2}\,dx$ can be intrepreted as the area of this quarter circle. $A=\frac{9\pi}{4}$.

6. C: $\int_1^4\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)^2dx=\int_1^4\left(x+2+\frac{1}{x}\right)dx$
$=\left(\frac{x^2}{2}+2x+\ln x\right)\Big|_1^4=\frac{27}{2}+\ln 4$

7. C: $\lim_{x\to\infty}\arccos\left(\frac{1-x}{2x+3}\right)=\arccos\left(\lim_{x\to\infty}\frac{1-x}{2x+3}\right)$
$=\arccos\left(-\frac{1}{2}\right)=\frac{2\pi}{3}$

8. B: We call $x=c$ a critical number if $x=c$ is in the domain of f and either $f'(c)=0$ or $f'(c)$ does not exist. The domain of $f(x)=\sqrt[3]{x^2-2x}$ is all real numbers, hence we need to solve $f'(c)=0$ or $f'(c)$ does not exist. Now $f(x)=(x^2-2x)^{1/3}$, so by the chain rule, $f'(x)=1/3(x^2-2x)^{-2/3}(2x-2)=\frac{2x-2}{3\sqrt[3]{(x^2-2x)^2}}$. So $f'(x)=0$ if $x=1$ and $f'(x)$ does not exist if $x=0$ or $x=2$.

9. D: First we will find the critical numbers for
$f(x)=x^3-12x+1$ on $[-3,0]$. $f'(x)=3x^2-12$, so $f'(x)=0$ for $x=\pm 2$. Since $x=2$ is not in the interval $[-3,0]$, we will only consider the critical number $x=-2$. Now, $f(-2)=17$, $f(-3)=10$ and $f(0)=1$. Hence the absolute maximum is 17 and the absolute minimum is 1.

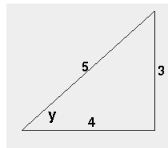
10. C: $f(x)=x^2e^x$, so by the product rule,
$f'(x)=2xe^x+x^2e^x=xe^x(2+x)$. The critcial numbers are therefore $x=0$ and $x=-2$. Now, $f'(x)>0$ for $x<-2$, $f'(x)<0$ for $-2<x<0$ and $f'(x)>0$ for $x>0$. Hence $f(x)$ has a local maximum at $x=-2$ and a local minimum at $x=0$.

11. D: Since $n=4$, $\Delta x=\frac{2-1}{4}=\frac{1}{4}$. Dividing the interval $[1,2]$ into 4 intervals of equal width yields the partition $\{1,5/4,3/2,7/4,2\}$. The midpoints of each subinterval are $\overline{x}_1=9/8$,
$\overline{x}_2=11/8$, $\overline{x}_3=13/8$, $\overline{x}_4=15/8$. Thus
$\int_1^2\ln x\,dx\approx\frac{1}{4}\left(\ln\frac{9}{8}+\ln\frac{11}{8}+\ln\frac{13}{8}+\ln\frac{15}{8}\right)$

12. C: By part I of the Fundamental Theorem of Calculus and the chain rule,
$\frac{d}{dx}\int_1^{\ln x}\frac{dt}{\sqrt{t^3+t}}=\frac{1}{\sqrt{(\ln x)^3+\ln x}}\frac{d}{dx}(\ln x)$
$\frac{1}{x\sqrt{(\ln x)^3+\ln x}}$

13. $\lim_{x\to\infty}\left(1-\frac{2}{x}\right)^x$ is of the form 1^∞.
Let $y=\left(1-\frac{2}{x}\right)^x$. Then $\ln y=x\ln\left(1-\frac{2}{x}\right)$.
Now $\lim_{x\to\infty}\ln y=\lim_{x\to\infty}x\ln\left(1-\frac{2}{x}\right)$
$=\lim_{x\to\infty}\frac{\ln\left(1-\frac{2}{x}\right)}{\frac{1}{x}}$. This limit is of the form $\frac{0}{0}$, so we will use L'Hospital's rule.
$\lim_{x\to\infty}\frac{\ln\left(1-\frac{2}{x}\right)}{\frac{1}{x}}=\lim_{x\to\infty}\frac{\frac{2/x^2}{1-2/x}}{-1/x^2}=\lim_{x\to\infty}\frac{-2}{1-\frac{2}{x}}=$
-2. Hence $\lim_{x\to\infty}\left(1-\frac{2}{x}\right)^x=e^{-2}$.

14. (a) To find $\sin\left(\arccos\left(\frac{4}{5}\right)\right)$, first let $y = \arccos\left(\frac{4}{5}\right)$. Then $\cos y = \frac{4}{5}$. By viewing the



triangle below, we see that $\sin x = \frac{3}{5}$.

(b) $f(x) = \arctan(\arcsin x)$, so by the chain rule, $f'(x) = \frac{1}{1+(\arcsin x)^2}\frac{1}{\sqrt{1-x^2}}$.

15. $\mathbf{a}(t) = \langle 1, 2t\rangle$, initial velocity $\mathbf{v}(0) = \langle 1, -1\rangle$ and initial position $\mathbf{s}(0) = \langle 0, 1\rangle$. Antidifferentiate $\mathbf{a}(t) = \langle 1, 2t\rangle$ and resolve the constants to find $\mathbf{v}(t)$. $\mathbf{v}(t) = \langle t+1, t^2-1\rangle$. Antidifferentiate $\mathbf{v}(t) = \langle t+1, t^2-1\rangle$ and resolve the constants to find $\mathbf{s}(t)$. $\mathbf{s}(t) = \langle t^2/2 + t, t^3/3 - t + 1\rangle$.

16. We are given the surface area of the box is 36 square feet. Let w be the width of the box and h the height of the box. Since the length of the base is 3 times the width, $l = 3w$. Now, the total surface area of the box is the area of the top and bottom and the area of the 4 sides. Thus $SA = 2(3w^2) + 2wh + 2(3wh) = 6w^2 + 8wh$. Since $SA = 36$, $6w^2 + 8wh = 36$. Solve this for h gives $h = \frac{18-3w^2}{4w}$. Now, we wish to maximize the volume. $V = 3w^2h$. Substitute $h = \frac{18-3w^2}{4w}$ yields $V = 3w^2(\frac{18-3w^2}{4w})$, thus $V = \frac{27}{2}w - \frac{9}{4}w^3$. Differentiate with respect to w. $V' = \frac{27}{2} - \frac{27}{4}w^2$. $V' = 0$ if $w = \sqrt{2}$. Now $V'' = -\frac{27}{2}w < 0$ if $w = \sqrt{2}$. Thus $\sqrt{2}$ yields a maximum by the second derivative test. The dimensions of the box are $w = \sqrt{2}$ ft, $l = 3\sqrt{2}$ ft and $h = \frac{3}{\sqrt{2}}$ ft.

17. a.) $f(x) = x^4 - 6x^2 + 4$, thus $f'(x) = 4x^3 - 12x = 4x(x^2-3)$. This yields critical numbers of $x = 0$ and $x = \pm\sqrt{3}$. Now $f'(x) < 0$ for $x < \sqrt{3}$ and $0 < x < \sqrt{3}$ and $f'(x) > 0$ for $-\sqrt{3} < x < 0$ and $x > \sqrt{3}$, hence $f(x)$ is increasing for $-\sqrt{3} < x < 0$ and $x > \sqrt{3}$ and $f(x)$ is decreasing for $x < -\sqrt{3}$ and $0 < x < \sqrt{3}$.

b.) $f''(x) = 12x^2 - 12 = 12(x^2-1)$. Thus $f''(x) = 0$ when $x = \pm 1$. $f''(x) > 0$ for $x < -1$ and $x > 1$ and $f''(x) < 0$ for $-1 < x < 1$. Hence $f(x)$ is concave up for $x < -1$ and $x > 1$ and $f(x)$ is concave down for $-1 < x < 1$.

c.) Since $f(x)$ changes concavity at $x = \pm 1$, these are inflection points.