

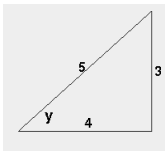
Math 151 Fall 2008 Exam III Solutions-Form B

1. C: $\lim_{x \rightarrow \infty} \arccos\left(\frac{1-x}{2x+3}\right) = \arccos\left(\lim_{x \rightarrow \infty} \frac{1-x}{2x+3}\right)$
 $= \arccos\left(-\frac{1}{2}\right) = \frac{2\pi}{3}$
2. C: $f'(x) = 3 \cos x + 5 \sin x$. Antidifferentiating,
 $f(x) = 3 \sin x - 5 \cos x + C$. Now, we are given that
 $f(0) = 4$, so $4 = 3 \sin(0) - 5 \cos(0) + C$, hence
 $C = 9$. Thus $f(x) = 3 \sin x - 5 \cos x + 9$, and there-
fore $f(\pi) = 14$.
3. B: We call $x = c$ a critical number of f if $x = c$ is in
the domain of f and either $f'(c) = 0$ or $f'(c)$ does
not exist. The domain of $f(x) = \sqrt[3]{x^2 - 2x}$ is all
real numbers, hence we need to solve $f'(c) = 0$ or
 $f'(c)$ does not exist. Now $f(x) = (x^2 - 2x)^{1/3}$, so by
the chain rule, $f'(x) = 1/3(x^2 - 2x)^{-2/3}(2x - 2) =$
 $\frac{2x - 2}{3\sqrt[3]{(x^2 - 2x)^2}}$. So $f'(x) = 0$ if $x = 1$ and $f'(x)$ does
not exist if $x = 0$ or $x = 2$.
4. D: First we will find the critical numbers for
 $f(x) = x^3 - 12x + 1$ on $[-3, 0]$. $f'(x) = 3x^2 - 12$,
so $f'(x) = 0$ for $x = \pm 2$. Since $x = 2$ is not in the
interval $[-3, 0]$, we will only consider the critical
number $x = -2$. Now, $f(-2) = 17$, $f(-3) = 10$
and $f(0) = 1$. Hence the absolute maximum is 17
and the absolute minimum is 1.
5. D: $\lim_{x \rightarrow 0} \frac{2^x - 5^x}{x}$ is of the form $\frac{0}{0}$, so we will apply
L'Hospital's rule.
 $\lim_{x \rightarrow 0} \frac{2^x - 5^x}{x} = \lim_{x \rightarrow 0} \frac{2^x \ln 2 - 5^x \ln 5}{1} = \ln 2 - \ln 5$
 $= \ln \frac{2}{5}$
6. C: $f(x) = x^2 e^x$, so by the product rule,
 $f'(x) = 2x e^x + x^2 e^x = x e^x (2 + x)$. The critical
numbers are therefore $x = 0$ and $x = -2$. Now,
 $f'(x) > 0$ for $x < -2$, $f'(x) < 0$ for $-2 < x < 0$
and $f'(x) > 0$ for $x > 0$. Hence $f(x)$ has a local
maximum at $x = -2$ and a local minimum at $x = 0$.
7. E: Note that $y = \sqrt{9 - x^2}$ for $0 \leq x \leq 3$ is a quarter
of the circle $x^2 + y^2 = 9$ in the first quadrant. Thus
 $\int_0^3 \sqrt{9 - x^2} dx$ can be interpreted as the area of this
quarter circle. $A = \frac{9\pi}{4}$.

8. D: Since $n = 4$, $\Delta x = \frac{2-1}{4} = \frac{1}{4}$. Dividing the
interval $[1, 2]$ into 4 intervals of equal width yields
the partition $\{1, 5/4, 3/2, 7/4, 2\}$. The midpoints of
each subinterval are $\bar{x}_1 = 9/8$,
 $\bar{x}_2 = 11/8$, $\bar{x}_3 = 13/8$, $\bar{x}_4 = 15/8$. Thus
 $\int_1^2 \ln x dx \approx \frac{1}{4} \left(\ln \frac{9}{8} + \ln \frac{11}{8} + \ln \frac{13}{8} + \ln \frac{15}{8} \right)$
9. D: $y = y_0 e^{kt}$. Since the population triples every 5
minutes, $y(5) = 3y_0$. Thus $3y_0 = y_0 e^{5k}$. Solve for
 k : $3 = e^{5k}$, thus $\ln 3 = \ln e^{5k}$, yielding $\frac{\ln 3}{5} = k$.
10. C: By part I of the Fundamental Theorem of Cal-
culus and the chain rule,
 $\frac{d}{dx} \int_1^{\ln x} \frac{dt}{\sqrt{t^3 + t}} = \frac{1}{\sqrt{(\ln x)^3 + \ln x}} \frac{d}{dx} (\ln x)$
 $= \frac{1}{x \sqrt{(\ln x)^3 + \ln x}}$
11. B: $f(x) = \ln(x + \ln x)$. By the chain rule,
 $f'(x) = \frac{1 + 1/x}{x + \ln x}$. Thus $f'(e) = \frac{1 + 1/e}{e + \ln e} = \frac{1}{e}$.
12. C: $\int_1^4 \left(\sqrt{x} + \frac{1}{\sqrt{x}} \right)^2 dx = \int_1^4 \left(x + 2 + \frac{1}{x} \right) dx$
 $= \left(\frac{x^2}{2} + 2x + \ln x \right) \Big|_1^4 = \frac{27}{2} + \ln 4$
13. $\lim_{x \rightarrow \infty} \left(1 - \frac{3}{x} \right)^x$ is of the form 1^∞ .
Let $y = \left(1 - \frac{3}{x} \right)^x$. Then $\ln y = x \ln \left(1 - \frac{3}{x} \right)$.
Now $\lim_{x \rightarrow \infty} \ln y = \lim_{x \rightarrow \infty} x \ln \left(1 - \frac{3}{x} \right)$
 $= \lim_{x \rightarrow \infty} \frac{\ln \left(1 - \frac{3}{x} \right)}{\frac{1}{x}}$. This limit is of the form $\frac{0}{0}$, so
we will use L'Hospital's rule.
 $\lim_{x \rightarrow \infty} \frac{\ln \left(1 - \frac{3}{x} \right)}{\frac{1}{x}} = \lim_{x \rightarrow \infty} \frac{\frac{3/x^2}{1-2/x}}{-1/x^2} = \lim_{x \rightarrow \infty} \frac{-3}{1 - \frac{2}{x}} =$
 -3 . Hence $\lim_{x \rightarrow \infty} \left(1 - \frac{2}{x} \right)^x = e^{-3}$.

14. (a) To find $\sin\left(\arccos\left(\frac{4}{5}\right)\right)$, first let

$y = \arccos\left(\frac{4}{5}\right)$. Then $\cos y = \frac{4}{5}$. By viewing the



triangle below, we see that $\sin x = \frac{3}{5}$.

(b) $f(x) = \arctan(\arcsin x)$, so by the chain rule,

$$f'(x) = \frac{1}{1 + (\arcsin x)^2} \frac{1}{\sqrt{1 - x^2}}.$$

15. $\mathbf{a}(t) = \langle 1, 2t \rangle$, initial velocity $\mathbf{v}(0) = \langle 1, -1 \rangle$ and initial position $\mathbf{s}(0) = \langle 0, 1 \rangle$. Antidifferentiate $\mathbf{a}(t) = \langle 1, 2t \rangle$ and resolve the constants to find $\mathbf{v}(t)$. $\mathbf{v}(t) = \langle t + 1, t^2 - 1 \rangle$. Antidifferentiate $\mathbf{v}(t) = \langle t + 1, t^2 - 1 \rangle$ and resolve the constants to find $\mathbf{s}(t)$. $\mathbf{s}(t) = \langle t^2/2 + t, t^3/3 - t + 1 \rangle$.

16. We are given the surface area of the box is 36 square feet. Let w be the width of the box and h the height of the box. Since the length of the base is 3 times the width, $l = 3w$. Now, the total surface area of the box is the area of the top and bottom and the area of the 4 sides. Thus $SA = 2(3w^2) + 2wh + 2(3wh) = 6w^2 + 8wh$. Since $SA = 36$, $6w^2 + 8wh = 36$. Solve this for h gives $h = \frac{18 - 3w^2}{4w}$. Now, we wish to maximize the volume. $V = 3w^2h$. Substitute $h = \frac{18 - 3w^2}{4w}$ yields $V = 3w^2\left(\frac{18 - 3w^2}{4w}\right)$, thus $V = \frac{27}{2}w - \frac{9}{4}w^3$. Differentiate with respect to w . $V' = \frac{27}{2} - \frac{27}{4}w^2$. $V' = 0$ if $w = \sqrt{2}$. Now $V'' = -\frac{27}{2}w < 0$ if $w = \sqrt{2}$. Thus $\sqrt{2}$ yields a maximum by the second derivative test. The dimensions of the box are $w = \sqrt{2}$ ft, $l = 3\sqrt{2}$ ft and $h = \frac{3}{\sqrt{2}}$ ft.

17. a.) $f(x) = x^4 - 6x^2 + 4$, thus $f'(x) = 4x^3 - 12x = 4x(x^2 - 3)$. This yields critical numbers of $x = 0$ and $x = \pm\sqrt{3}$. Now $f'(x) < 0$ for $x < \sqrt{3}$ and $0 < x < \sqrt{3}$ and $f'(x) > 0$ for $-\sqrt{3} < x < 0$ and $x > \sqrt{3}$, hence $f(x)$ is increasing for $-\sqrt{3} < x < 0$ and $x > \sqrt{3}$ and $f(x)$ is decreasing for $x < -\sqrt{3}$ and $0 < x < \sqrt{3}$.

b.) $f''(x) = 12x^2 - 12 = 12(x^2 - 1)$. Thus $f''(x) = 0$ when $x = \pm 1$. $f''(x) > 0$ for $x < -1$ and $x > 1$ and $f''(x) < 0$ for $-1 < x < 1$. Hence $f(x)$ is concave

up for $x < -1$ and $x > 1$ and $f(x)$ is concave down for $-1 < x < 1$.

c.) Since $f(x)$ changes concavity at $x = \pm 1$, these are inflection points.