MATH 151, SPRING SEMESTER 2009
COMMON EXAMINATION I - VERSION A

Name (print):____________________________________

Signature: ____________________________________

Instructor's name: ____________________________________

Section No: ____________________________________

INSTRUCTIONS

1. In Part 1 (Problems 1–11), mark your responses on your ScanTron form using a No: 2 pencil. *For your own record, mark your choices on the exam as well.*

2. Calculators **should not be used** throughout the examination.

3. In Part 2 (Problems 12–16), present your solutions in the space provided. **Show all your work** neatly and concisely, and **indicate your final answer clearly**. You will be graded, not merely on the final answer, but also on the quality and correctness of the work leading up to it.

4. Be sure to **write your name, section number, and version letter of the exam on the ScanTron form**.

Part 1 – Multiple Choice (44 points)

Each question is worth **4 points**. *Mark your responses on the ScanTron form and on the exam itself.*

1. If $\mathbf{a} = \langle 2, 3\rangle$ and $\mathbf{b} = \langle 1, -2\rangle$, compute $\mathbf{a} - 2\mathbf{b}$.

(a) $\langle 3, 8\rangle$

(b) $\langle 0, -1\rangle$

(c) $\langle 0, 7\rangle$

(d) $\langle 3, 4\rangle$

(e) $\langle 1, 5\rangle$

2. Suppose that s and t are real numbers, and let $\mathbf{a} = \langle s, 2\rangle$ and $\mathbf{b} = \langle -1, t\rangle$. Given that $\mathbf{a}$ is orthogonal to $\mathbf{b}$, what is the relationship between s and t?

(a) $t = 2s$

(b) $s = t$

(c) $s = -t$

(d) the vectors are always orthogonal, regardless of the values of s and t

(e) $s = 2t$

3. Determine a vector equation of the straight line which passes through the point $(1, -1)$ and is parallel to the vector $\langle -2, 3\rangle$.

(a) $\mathbf{r}(t) = \langle -2 + t, 3 - t\rangle$

(b) $\mathbf{r}(t) = \langle -2 - t, 3 + t\rangle$

(c) $\mathbf{r}(t) = \langle t, -t\rangle$

(d) $\mathbf{r}(t) = \langle -1 - 2t, 1 + 3t\rangle$

(e) $\mathbf{r}(t) = \langle 1 - 2t, -1 + 3t\rangle$

4. Find a unit vector parallel to $2\mathbf{i} - \mathbf{j}$.

(a) $\sqrt{5}(2\mathbf{i} - \mathbf{j})$

(b) $\sqrt{5}(\mathbf{i} - 2\mathbf{j})$

(c) $\dfrac{1}{\sqrt{5}}(2\mathbf{i} - \mathbf{j})$

(d) $\dfrac{1}{\sqrt{5}}(\mathbf{i} + 2\mathbf{j})$

(e) $\dfrac{1}{\sqrt{5}}(\mathbf{i} - 2\mathbf{j})$

5. Find a Cartesian equation for the parametric curve given by the equations

$$x(t) = \frac{\cos t}{2}, \quad y(t) = \sin t, \qquad 0 \le t \le 2\pi.$$

(a) $\dfrac{x^2}{4} + y^2 = 1$

(b) $x^2 + 4y^2 = 1$

(c) $x^2 + \dfrac{y^2}{4} = 1$

(d) $4x^2 + y^2 = 1$

(e) $x^2 + y^2 = 2$

6. Compute $\lim\limits_{x \to 3} \dfrac{x^2 - 5x + 6}{x^2 - 9}$.

(a) $5/6$
(b) 1
(c) $-2/3$
(d) $1/6$
(e) does not exist

7. Compute $\lim\limits_{x\to\infty} \dfrac{3x+5}{\sqrt{4x^2+x+1}}$.

(a) $3/2$
(b) $3/4$
(c) $5/2$
(d) $5/4$
(e) does not exist

8. Differentiate the function $f(x) = 2\sqrt{x} + \dfrac{3}{\sqrt{x}}$ with respect to x.

(a) $2x^{-1/2} - 3x^{-3/2}$

(b) $3x^{-1/2} - 2x^{-3/2}$

(c) $\dfrac{3}{2}\, x^{-1/2} - x^{-3/2}$

(d) $x^{-1/2} + 6x^{1/2}$

(e) $x^{-1/2} - \dfrac{3}{2}\, x^{-3/2}$

9. The function f, defined by the equations

$$f(x) = \begin{cases} 2, & \text{if } x < 0; \\ |x-1|, & \text{if } 0 \le x \le 2; \\ 1, & \text{if } x > 2, \end{cases}$$

is continuous, but not differentiable at

(a) $x = 0, 1$
(b) $x = 0, 2$
(c) $x = 0$
(d) $x = 1, 2$
(e) $x = 0, 1, 2$

10. Suppose that f is a differentiable function and that $g(x) = (x^3 + 2x)f(x)$. Given that $f(1) = f'(1) = 1$, calculate the value of $g'(1)$.

(a) 8
(b) 5
(c) 1
(d) insufficient information to make a determination
(e) 0

11. Suppose that f is a differentiable function satisfying the following conditions: $f(1) = 1$, and $(x^2 + 1)f'(x) - 2xf(x) = x(2 - x)$ for every real number x. Determine the slope of the tangent to the graph of $y = f(x)$ at the point $(1, 1)$.

(a) 0
(b) 3/2
(c) 1
(d) insufficient information to make a determination
(e) 1/2

Part 2 (60 points)

Present your solutions to the following problems (12–16) in the space provided. **Show all your work** *neatly and concisely, and* **indicate your final answer clearly**. *You will be graded, not merely on the final answer, but also on the quality and correctness of the work leading up to it.*

12. Let ABC be the triangle with vertices at the points $A(1,2)$, $B(6,1)$, and $C(-1,-2)$.

(i) (`8 points`) Find the vectors $\vec{AB}$ and $\vec{AC}$, and compute their magnitudes.

(ii) (`6 points`) Calculate the cosine of the angle at the vertex A of the triangle.

13. (`10 points`) Let

$$\mathbf{a} = 2\mathbf{i} - 3\mathbf{j} \quad \text{and} \quad \mathbf{b} = \mathbf{i} + 6\mathbf{j}.$$

Compute the scalar projection of $\mathbf{b}$ onto $\mathbf{a}$ ($\text{comp}_{\mathbf{a}}\mathbf{b}$) and the vector projection of $\mathbf{b}$ onto $\mathbf{a}$ ($\text{proj}_{\mathbf{a}}\mathbf{b}$).

14. (`12 points`) Let A be a constant, and consider the function

$$f(x) = \begin{cases} 4x + A, & \text{if } x < 2; \\ 2, & \text{if } x = 2; \\ x^2 - Ax + 1, & \text{if } x > 2. \end{cases}$$

Determine the value of A for which $\lim_{x \to 2} f(x)$ exists.

15. (`12 points`) Let

$$f(x) = \sqrt{2x+3}\,, \qquad x \geq -3/2.$$

Compute the value of $f'(1)$ *using the definition of the derivative*. (Note: No credit will be given for using any other method, correct answer notwithstanding.)

16. (`12 points`) Let C denote the graph of the function

$$f(x) = \frac{2x}{x+2}, \qquad x \neq -2.$$

Determine all points on C where the tangent to C is parallel to the line $y = x$.

QN	**PTS**
1–11	
12	
13	
14	
15	
16	
TOTAL	