Exam I Solutions (Form A)

1. **C** $\mathbf{a} - 2\mathbf{b} =< 2, 3 > - < 2, -4 >=< 0, 7 >$

2. **E** $\mathbf{a}$ is orthogonal to $\mathbf{b}$ mean $\mathbf{a} \cdot \mathbf{b} = 0$, so $(s)(-1) + 2t = 0$, or $s = 2t$

3. **E** The equation of the line is given by $\mathbf{r}(t) = \mathbf{r_0} + t\mathbf{v}$,where $\mathbf{r_0} =< 1, -1 >$and $\mathbf{v} =< -2, 3 >$, so the equation is $\mathbf{r}(t) =< 1, -1 > +t < -2, 3 >$, or $\mathbf{r}(t) =< 1 - 2t, -1 + 3t >$

4. **C** To find a parallel unit vector, multiply the given vector by the reciprocal of its magnitude. The result is $\dfrac{1}{\sqrt{2^2 + (-1)^2}}(2\mathbf{i} - \mathbf{j}) = \dfrac{1}{\sqrt{5}}(2\mathbf{i} - \mathbf{j})$

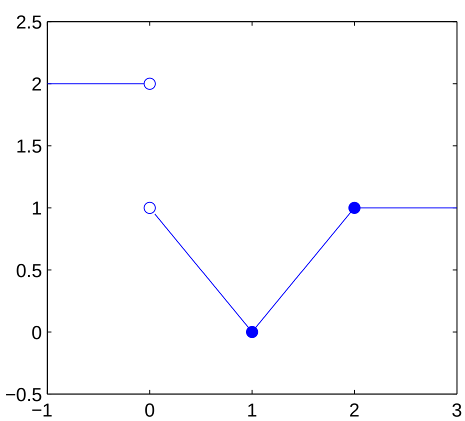
5. **D** The Cartesian equation is based on the identity $\cos^2 t + \sin^2 t = 1$. Since $\cos t = 2x$ and $\sin t = y$, the identity becomes $(2x)^2 + y^2 = 1$, or $4x^2 + y^2 = 1$

6. **D** Factor and cancel: $\displaystyle\lim_{x \to 3} \frac{(x-2)(x-3)}{(x+3)(x-3)} = \lim_{x \to 3} \frac{x-2}{x+3} = \frac{1}{6}$

7. **A** Factor out the dominating terms, i.e., highest powers of x. (For the denominator, $\sqrt{x^2} = |x| = x$ when x is positive): $\displaystyle\lim_{x \to \infty} \frac{x(3 + \frac{5}{x})}{x\sqrt{4 + \frac{1}{x} + \frac{1}{x^2}}} = \lim_{x \to \infty} \frac{3 + \frac{5}{x}}{\sqrt{4 + \frac{1}{x} + \frac{1}{x^2}}} = \frac{3}{2}$.

8. **E** $f(x) = 2x^{1/2} + 3x^{-1/2}$, so $f'(x) = x^{-1/2} - \dfrac{3}{2}x^{-3/2}$.

9. **D** The function is graphed below. From the graph, it is clear that f is continuous, but not differentiable at $x = 1, 2$



10. **A** Use the product rule, then substitute 1: $g'(x) = (x^3 + 2x)f'(x) + f(x)(3x^2 + 2)$, $g'(1) = (1 + 2)f'(1) + f(1)(3 + 2)$. Since $f(1) = f'(1) = 1$, $g'(1) = 3(1) + 1(5) = 8$.

11. **B** We are looking for $f'(1)$. Since the given equation is true for every real number x,substitute $x = 1$: $(1^2 + 1)f'(1) - 2(1)f(1) = 1(2 - 1)$; $2f'(1) - 2f(1) = 1$. Since $f(1) = 1$,substitute and solve for $f'(1)$: $2f'(1) - 2 = 1$, $f'(1) = \dfrac{3}{2}$.

12. i) $\mathbf{AB} =< 6, 1 > - < 1, 2 >=< 5, -1 >$, $|\mathbf{AB}| = \sqrt{5^2 + (-1)^2} = \sqrt{26}$. $\mathbf{AC} =< -1, -2 > - < 1, 2 >=< -2, -4 >$, $|\mathbf{AC}| = \sqrt{(-2)^2 + (-4)^2} = \sqrt{20}$
    ii) $\cos A = \dfrac{\mathbf{AB} \cdot \mathbf{AC}}{|\mathbf{AB}||\mathbf{AC}|} = \dfrac{(5)(-2) + (-1)(-4)}{\sqrt{26}\sqrt{20}} = \dfrac{-6}{\sqrt{520}}$

13. $comp_{\mathbf{a}}\mathbf{b} = \dfrac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}|} = \dfrac{(2)(1)+(-3)(6)}{\sqrt{2^2+(-3)^2}} = \dfrac{-16}{\sqrt{13}}$

$\mathbf{proj_a b} = \dfrac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}|^2}\mathbf{a} = \dfrac{-16}{13}(2\mathbf{i}-3\mathbf{j}) = \dfrac{-32}{13}\mathbf{i} + \dfrac{48}{13}\mathbf{j}$

14. For the limit to exist, we must have $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x)$.
$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} 4x + A = 8 + A$.
$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} x^2 - Ax + 1 = 4 - 2A + 1$. Equating these gives us $8+A = 5-2A$,or $A=-1$.
(NOTE that in calculating the limit, we do not care about the value at $x=2$)

15. $f'(1) = \lim_{x\to 1}\dfrac{f(x)-f(1)}{x-1} = \lim_{x\to 1}\dfrac{\sqrt{2x+3}-\sqrt{5}}{x-1}$. Rationalize the numerator using the conjugate:
$f'(1) = \lim_{x\to 1}\dfrac{(2x+3)-5}{(x-1)(\sqrt{2x+3}+\sqrt{5})} = \lim_{x\to 1}\dfrac{2x-2}{(x-1)(\sqrt{2x+3}+\sqrt{5})} = \lim_{x\to 1}\dfrac{2}{\sqrt{2x+3}+\sqrt{5}} = \dfrac{2}{2\sqrt{5}} = \dfrac{1}{\sqrt{5}}$.

16. We want the values of x where $f'(x)=1$. $f'(x) = \dfrac{(x+2)(2)-(2x)(1)}{(x+2)^2} = 1$, $\dfrac{2x+4-2x}{x^2+4x+4} = 1$, $4 = x^2+4x+4$, $0 = x^2+4x$, $0 = x(x+4)$,so $x=0$ and $x=-4$. Evaluating f at these values leads to the points $(0,0)$ and $(-4,4)$. NOTE: It turns out that the tangent line at $(0,0)$ is technically not parallel to $y=x$, but coincides with it since the point $(0,0)$ is on this line.