

**MATH 151, SPRING SEMESTER 2009  
COMMON EXAMINATION I - VERSION B**

Name (print): \_\_\_\_\_

Signature: \_\_\_\_\_

Instructor's name: \_\_\_\_\_

Section No: \_\_\_\_\_

**INSTRUCTIONS**

1. In Part 1 (Problems 1–11), mark your responses on your ScanTron form using a No. 2 pencil. *For your own record, mark your choices on the exam as well.*
2. Calculators **should not be used** throughout the examination.
3. In Part 2 (Problems 12–16), present your solutions in the space provided. **Show all your work** neatly and concisely, and **indicate your final answer clearly**. You will be graded, not merely on the final answer, but also on the quality and correctness of the work leading up to it.
4. Be sure to **write your name, section number, and version letter of the exam on the ScanTron form.**

**Part 1 – Multiple Choice (44 points)**

Each question is worth **4 points**. Mark your responses on the ScanTron form and on the exam itself.

1. If  $\mathbf{a} = \langle 2, 3 \rangle$  and  $\mathbf{b} = \langle 1, -2 \rangle$ , compute  $2\mathbf{a} - \mathbf{b}$ .
  - (a)  $\langle 3, 8 \rangle$
  - (b)  $\langle 0, -1 \rangle$
  - (c)  $\langle 0, 7 \rangle$
  - (d)  $\langle 3, 4 \rangle$
  - (e)  $\langle 1, 5 \rangle$
2. Suppose that  $s$  and  $t$  are real numbers, and let  $\mathbf{a} = \langle t, 2 \rangle$  and  $\mathbf{b} = \langle -1, s \rangle$ . Given that  $\mathbf{a}$  is orthogonal to  $\mathbf{b}$ , what is the relationship between  $s$  and  $t$ ?
  - (a)  $t = 2s$
  - (b)  $s = t$
  - (c)  $s = -t$
  - (d) the vectors are always orthogonal, regardless of the values of  $s$  and  $t$
  - (e)  $s = 2t$
3. Determine a vector equation of the straight line which passes through the point  $(-1, 1)$  and is parallel to the vector  $\langle -2, 3 \rangle$ .
  - (a)  $\mathbf{r}(t) = \langle -2 + t, 3 - t \rangle$
  - (b)  $\mathbf{r}(t) = \langle -2 - t, 3 + t \rangle$
  - (c)  $\mathbf{r}(t) = \langle t, -t \rangle$
  - (d)  $\mathbf{r}(t) = \langle -1 - 2t, 1 + 3t \rangle$
  - (e)  $\mathbf{r}(t) = \langle 1 - 2t, -1 + 3t \rangle$

4. Find a unit vector parallel to  $\mathbf{i} - 2\mathbf{j}$ .

(a)  $\sqrt{5}(2\mathbf{i} - \mathbf{j})$

(b)  $\sqrt{5}(\mathbf{i} - 2\mathbf{j})$

(c)  $\frac{1}{\sqrt{5}}(2\mathbf{i} - \mathbf{j})$

(d)  $\frac{1}{\sqrt{5}}(2\mathbf{i} + \mathbf{j})$

(e)  $\frac{1}{\sqrt{5}}(\mathbf{i} - 2\mathbf{j})$

5. Find a Cartesian equation for the parametric curve given by the equations

$$x(t) = \cos t, \quad y(t) = \frac{\sin t}{2}, \quad 0 \leq t \leq 2\pi.$$

(a)  $\frac{x^2}{4} + y^2 = 1$

(b)  $x^2 + 4y^2 = 1$

(c)  $x^2 + \frac{y^2}{4} = 1$

(d)  $4x^2 + y^2 = 1$

(e)  $x^2 + y^2 = 2$

6. Compute  $\lim_{x \rightarrow 3} \frac{x^2 - x - 6}{x^2 - 9}$ .

(a)  $5/6$

(b)  $1$

(c)  $-2/3$

(d)  $1/6$

(e) does not exist

7. Compute  $\lim_{x \rightarrow \infty} \frac{5x + 3}{\sqrt{4x^2 + x + 1}}$ .

- (a)  $3/2$
- (b)  $3/4$
- (c)  $5/2$
- (d)  $5/4$
- (e) does not exist

8. Differentiate the function  $f(x) = 3\sqrt{x} + \frac{2}{\sqrt{x}}$  with respect to  $x$ .

- (a)  $2x^{-1/2} - 3x^{-3/2}$
- (b)  $3x^{-1/2} - 2x^{-3/2}$
- (c)  $\frac{3}{2}x^{-1/2} - x^{-3/2}$
- (d)  $x^{-1/2} + 6x^{1/2}$
- (e)  $x^{-1/2} - \frac{3}{2}x^{-3/2}$

9. The function  $f$ , defined by the equations

$$f(x) = \begin{cases} 2, & \text{if } x < 0; \\ |x - 1|, & \text{if } 0 \leq x \leq 2; \\ 1, & \text{if } x > 2, \end{cases}$$

is continuous, but not differentiable at

- (a)  $x = 0, 1$
- (b)  $x = 0, 2$
- (c)  $x = 0$
- (d)  $x = 1, 2$
- (e)  $x = 0, 1, 2$

10. Suppose that  $f$  is a differentiable function and that  $g(x) = (x^3 - 2x)f(x)$ . Given that  $f(1) = f'(1) = 1$ , calculate the value of  $g'(1)$ .
- (a) 8
  - (b) 5
  - (c) 1
  - (d) insufficient information to make a determination
  - (e) 0
11. Suppose that  $f$  is a differentiable function satisfying the following conditions:  $f(1) = 1$ , and  $(x^2 + 1)f'(x) - 2xf(x) = x(2 - x)$  for every real number  $x$ . Determine the slope of the tangent to the graph of  $y = f(x)$  at the point  $(1, 1)$ .
- (a) 0
  - (b)  $3/2$
  - (c) 1
  - (d) insufficient information to make a determination
  - (e)  $1/2$

**Part 2 (60 points)**

*Present your solutions to the following problems (12–16) in the space provided. Show all your work neatly and concisely, and indicate your final answer clearly. You will be graded, not merely on the final answer, but also on the quality and correctness of the work leading up to it.*

**12.** Let  $ABC$  be the triangle with vertices at the points  $A(1, 2)$ ,  $B(6, 1)$ , and  $C(-1, -2)$ .

(i) (8 points) Find the vectors  $\vec{BC}$  and  $\vec{BA}$ , and compute their magnitudes.

(ii) (6 points) Calculate the cosine of the angle at the vertex  $B$  of the triangle.

13. (10 points) Let

$$\mathbf{a} = 3\mathbf{i} - 2\mathbf{j} \quad \text{and} \quad \mathbf{b} = \mathbf{i} + 6\mathbf{j}.$$

Compute the scalar projection of  $\mathbf{b}$  onto  $\mathbf{a}$  ( $\text{comp}_{\mathbf{a}}\mathbf{b}$ ) and the vector projection of  $\mathbf{b}$  onto  $\mathbf{a}$  ( $\text{proj}_{\mathbf{a}}\mathbf{b}$ ).

14. (12 points) Let  $A$  be a constant, and consider the function

$$f(x) = \begin{cases} 4x - A, & \text{if } x < 2; \\ 2, & \text{if } x = 2; \\ x^2 + Ax + 1, & \text{if } x > 2. \end{cases}$$

Determine the value of  $A$  for which  $\lim_{x \rightarrow 2} f(x)$  exists.

15. (12 points) Let

$$f(x) = \sqrt{3x + 2}, \quad x \geq -2/3.$$

Compute the value of  $f'(1)$  *using the definition of the derivative*. (Note: No credit will be given for using any other method, correct answer notwithstanding.)

**16.** (12 points) Let  $C$  denote the graph of the function

$$f(x) = \frac{2x}{x-2}, \quad x \neq 2.$$

Determine all points on  $C$  where the tangent to  $C$  is parallel to the line  $y = -x$ .

**QN            PTS**

1-11

12

13

14

15

16

**TOTAL**