Exam I Solutions (Form B)

1. **A** $2\mathbf{a} - \mathbf{b} =< 4, 6 > - < 1, -2 >=< 3, 8 >$

2. **A** $\mathbf{a}$ is orthogonal to $\mathbf{b}$ mean $\mathbf{a} \cdot \mathbf{b} = 0$, so $(t)(-1) + 2s = 0$, or $t = 2s$

3. **D** The equation of the line is given by $\mathbf{r}(t) = \mathbf{r_0} + t\mathbf{v}$,where $\mathbf{r_0} =< -1, 1 >$and $\mathbf{v} =< -2, 3 >$, so the equation is $\mathbf{r}(t) =< -1, 1 > +t < -2, 3 >$, or $\mathbf{r}(t) =< -1 - 2t,\ 1 + 3t >$

4. **E** To find a parallel unit vector, multiply the given vector by the reciprocal of its magnitude. The result is $\dfrac{1}{\sqrt{1^2 + (-2)^2}}(\mathbf{i} - 2\mathbf{j}) = \dfrac{1}{\sqrt{5}}(\mathbf{i} - 2\mathbf{j})$

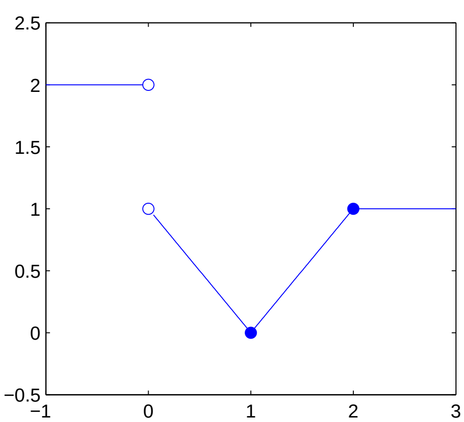
5. **B** The Cartesian equation is based on the identity $\cos^2 t + \sin^2 t = 1$. Since $\cos t = x$ and $\sin t = 2y$, the identity becomes $x^2 + (2y)^2 = 1$, or $x^2 + 4y^2 = 1$

6. **A** Factor and cancel: $\lim\limits_{x\to 3} \dfrac{(x+2)(x-3)}{(x+3)(x-3)} = \lim\limits_{x\to 3} \dfrac{x+2}{x+3} = \dfrac{5}{6}$

7. **C** Factor out the dominating terms, i.e., highest powers of x. (For the denominator, $\sqrt{x^2} = |x| = x$ when x is positive): $\lim\limits_{x\to\infty} \dfrac{x(5 + \frac{3}{x})}{x\sqrt{4 + \frac{1}{x} + \frac{1}{x^2}}} = \lim\limits_{x\to\infty} \dfrac{5 + \frac{3}{x}}{\sqrt{4 + \frac{1}{x} + \frac{1}{x^2}}} = \dfrac{5}{2}$.

8. **C** $f(x) = 3x^{1/2} + 2x^{-1/2}$, so $f'(x) = \dfrac{3}{2}x^{-1/2} - x^{-3/2}$.

9. **D** The function is graphed below. From the graph, it is clear that f is continuous, but not differentiable at $x = 1, 2$



10. **E** Use the product rule, then substitute 1: $g'(x) = (x^3 - 2x)f'(x) + f(x)(3x^2 - 2)$, $g'(1) = (1-2)f'(1) + f(1)(3-2)$. Since $f(1) = f'(1) = 1$, $g'(1) = (-1)(1) + 1(1) = 0$.

11. **B** We are looking for $f'(1)$. Since the given equation is true for every real number x,substitute $x = 1$: $(1^2 + 1)f'(1) - 2(1)f(1) = 1(2 - 1)$; $2f'(1) - 2f(1) = 1$. Since $f(1) = 1$,substitute and solve for $f'(1)$: $2f'(1) - 2 = 1$, $f'(1) = \dfrac{3}{2}$.

12. i) $\mathbf{BC} =< -1, -2 > - < 6, 1 >=< -7, -3 >$, $|\mathbf{BC}| = \sqrt{(-7)^2 + (-3)^2} = \sqrt{58}$. $\mathbf{BA} =< 1, 2 > - < 6, 1 >=< -5, 1 >$, $|\mathbf{BA}| = \sqrt{(-5)^2 + 1^2} = \sqrt{26}$
ii) $\cos B = \dfrac{\mathbf{BC} \cdot \mathbf{BA}}{|\mathbf{BC}||\mathbf{BA}|} = \dfrac{(-7)(-5) + (-3)(1)}{\sqrt{58}\sqrt{26}} = \dfrac{32}{\sqrt{1508}}$ (multiplication of the denominator is NOT necessary)

13. $comp_{\mathbf{a}}\mathbf{b} = \dfrac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}|} = \dfrac{(3)(1)+(-2)(6)}{\sqrt{3^2+(-2)^2}} = \dfrac{-9}{\sqrt{13}}$

   $\mathbf{proj_a b} = \dfrac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}|^2}\mathbf{a} = \dfrac{-9}{13}(3\mathbf{i}-2\mathbf{j}) = \dfrac{-27}{13}\mathbf{i} + \dfrac{18}{13}\mathbf{j}$

14. For the limit to exist, we must have $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x)$.
   $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} 4x - A = 8 - A$.
   $\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} x^2 + Ax + 1 = 4 + 2A + 1$. Equating these gives us $8 - A = 5 + 2A$,or $A = 1$.
   (NOTE that in calculating the limit, we do not care about the value at $x = 2$)

15. $f'(1) = \lim_{x\to 1} \dfrac{f(x)-f(1)}{x-1} = \lim_{x\to 1} \dfrac{\sqrt{3x+2}-\sqrt{5}}{x-1}$. Rationalize the numerator using the conjugate:
   $f'(1) = \lim_{x\to 1} \dfrac{(3x+2)-5}{(x-1)(\sqrt{3x+2}+\sqrt{5})} = \lim_{x\to 1} \dfrac{3x-3}{(x-1)(\sqrt{3x+2}+\sqrt{5})} = \lim_{x\to 1} \dfrac{3}{\sqrt{3x+2}+\sqrt{5}} = \dfrac{3}{2\sqrt{5}}$.

16. We want the values of x where $f'(x) = -1$. $f'(x) = \dfrac{(x-2)(2)-(2x)(1)}{(x-2)^2} = -1$, $\dfrac{2x-4-2x}{x^2-4x+4} = -1$, $-4 = -x^2+4x-4$, $0 = -x^2+4x$, $0 = -x(x-4)$,so $x = 0$ and $x = 4$. Evaluating f at these values leads to the points $(0,0)$ and $(4,4)$. NOTE: It turns out that the tangent line at $(0,0)$ is technically not parallel to $y = -x$, but coincides with it since the point $(0,0)$ is on this line.