Exam II Solutions (Form A)

1. **D** $\frac{d}{dx}(2\sin x - \cos x) = 2\cos x - (-\sin x) = 2\cos x + \sin x$

2. **E** $y' = \sec x \tan x - 1$; when $x = 0$, $y' = \sec 0 \tan 0 - 1 = -1$

3. **C** $f'(x) = 8(2x-1)^7(2)$ using the Chain Rule, so $f'(0) = 8(-1)^7(2) = -16$

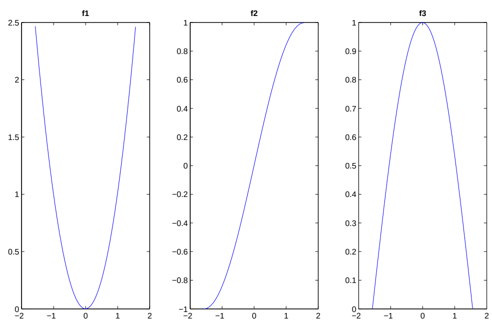
4. **B** The derivatives of $\sin x$ follow the pattern $\cos x$, $-\sin x$, $-\cos x$, $\sin x$, ... so the tenth derivative is the same as the second derivative, $-\sin x$.

5. **C** $\mathbf{r}'(t) = < 2e^{2t}, 1/3\, t^{-2/3} >$,so $\mathbf{r}'(1) = < 2e^2, 1/3 >$

6. **E** Newton's Method says that
$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 2 - \frac{2^3 - 2\cdot 2 - 5}{3(2^2) - 2} = 2 - \frac{-1}{10} = 21/10$

7. **B** Begin with the exponent: $\lim_{x\to 0^-} \frac{1}{x} = -\infty$. Thus $\lim_{x\to 0^-} 3^{1/x} = 0$ and $\lim_{x\to 0^-} \frac{2}{1+3^{(1/x)}} = \frac{2}{1+0} = 2$

8. **B** The graphs of the functions are shown below. Using the horizontal line test, f_1 and f_3 are NOT one-to-one.



9. **D** Either use the formula $L(x) = f(a) + f'(a)(x-a)$ or just find the equation of the tangent line. $f(\pi) = e^{\sin \pi} = e^0 = 1$. $f'(x) = e^{\sin x}\cos x$, so $f'(\pi) = e^{\sin \pi}\cos \pi = -1$. The equation is $L(x) = 1 + (-1)(x - \pi) = \pi + 1 - x$.

10. **A** Switch x and y and solve for y: $x = \frac{1}{2+y^2}$, $2x + xy^2 = 1$, $xy^2 = 1 - 2x$, $y = \sqrt{\frac{1-2x}{x}}$ (we take the positive square root since the domain of the original function is $[0, \infty)$). The domain of this function is when the fraction is positive, which occurs when $x \in (0, 1/2]$.

11. **A** The limit is nothing more than the definition of $f'(a)$, where $f(x) = \tan^2 x$ and $a = \frac{\pi}{4}$.
$f'(x) = 2\tan x \sec^2 x$, so $f'(\pi/4) = 2\tan\left(\frac{\pi}{4}\right)\sec^2\left(\frac{\pi}{4}\right) = 2(1)(\sqrt{2})^2 = 4$.

12. Using the Product Rule, $y' = 2xe^x + x^2e^x = (2x + x^2)e^x$, $y'' = (2+2x)e^x + (2x+x^2)e^x = (2+4x+x^2)e^x$, $y''' = (4+2x)e^x + (2+4x+x^2)e^x = (6+6x+x^2)e^x$.
Substituting all derivatives into the differential equation yields $(6+6x+x^2)e^x - 3(2+4x+x^2)e^x + 3(2x+x^2)e^x - x^2e^x = (6+6x+x^2-6-12x-3x^2+6x+3x^2-x^2)e^x = 0$

13. $y'\sqrt{x-1} + y(1/2)(x-1)^{-1/2} + \sqrt{y-1} + x(1/2)(y-1)^{-1/2}(y') = 2xy' + 2y$. Moving the terms with y' to the left and terms without y' to the right yields $y'(x-1)^{1/2} + (1/2)x(y-1)^{-1/2}y' - 2xy' = 2y - (1/2)y(x-1)^{-1/2} - (y-1)^{1/2}$, so $y' = \frac{2y - (1/2)y(x-1)^{-1/2} - (y-1)^{1/2}}{-2x + (1/2)x(y-1)^{-1/2} + (x-1)^{1/2}}$.

14. i) The relating formula is $V = x^3$, given $\dfrac{dV}{dt} = -2$. At the instant when $V = 27$,we know $x = 3$. Differentiate and substitute: $\dfrac{dV}{dt} = 3x^2\dfrac{dx}{dt}$, $-2 = 3(3)^2\dfrac{dx}{dt}$, or $\dfrac{dx}{dt} = -\dfrac{2}{27}\dfrac{m}{min}$.

ii) The relating formula is $S = 6x^2$. Differentiate: $\dfrac{dS}{dt} = 12x\dfrac{dx}{dt}$. At the instant when $x = 3$, $\dfrac{dx}{dt} = -\dfrac{2}{27}$, so substitute: , $\dfrac{dS}{dt} = 12(3)\left(-\dfrac{2}{27}\right) = -\dfrac{8}{3}\dfrac{m^2}{min}$.

15. i) Using the quotient rule $x'(t) = \dfrac{(1+t)(-1)-(1-t)(1)}{(1+t)^2} = \dfrac{-2}{(1+t)^2}$

ii) Using the quotient rule $y'(t) = \dfrac{(1+t)(1/2)t^{-1/2}-(t^{1/2}-1)(1)}{(1+t)^2}$

iii) The slope is found by $\dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt}$. When $x = 0$ and $y = 0$, we have $t = 1$, so the slope is $\left.\dfrac{dy}{dx}\right|_{t=1} = \dfrac{\frac{(1+1)(1/2)(1)^{-1/2}-(1^{1/2}-1)(1)}{(1+1)^2}}{\frac{-2}{(1+1)^2}} = -\dfrac{1}{2}$

16. The slope of the tangent line is given by $g'(1) = \dfrac{1}{f'(g(1))}$. $f'(x) = e^{x^3+2x}(3x^2+2)$. To find $g(1)$, solve $f(x) = 1$. $e^{x^3+2x} = 1$ when $x^3 + 2x = x(x^2+2) = 0$, or $x = 0$ (since $x^2 + 2 \neq 0$). So $g'(1) = \dfrac{1}{f'(0)} = \dfrac{1}{e^0(3\cdot 0+2)} = \dfrac{1}{2}$. Since $g(1) = 0$, the tangent line passes through the point $(1,0)$. Therefore, the equation is $y - 0 = \dfrac{1}{2}(x-1)$, or $y = \dfrac{1}{2}x - \dfrac{1}{2}$.