

**MATH 151, SPRING SEMESTER 2009  
COMMON EXAMINATION II - VERSION B**

Name (print): \_\_\_\_\_

Signature: \_\_\_\_\_

Instructor's name: \_\_\_\_\_

Section No: \_\_\_\_\_

**INSTRUCTIONS**

1. In Part 1 (Problems 1–11), mark your responses on your ScanTron form using a No: 2 pencil. *For your own record, mark your choices on the exam as well.* Collected scantrons will *not* be returned after the examination.
2. Calculators **should not be used** throughout the examination.
3. In Part 2 (Problems 12–16), present your solutions in the space provided. **Show all your work** neatly and concisely, and **indicate your final answer clearly**. You will be graded, not merely on the final answer, but also on the quality and correctness of the work leading up to it.
4. Be sure to **write your name, section number, and version letter of the exam on the ScanTron form.**

**Part 1 – Multiple Choice (44 points)**

*Each question is worth 4 points. Mark your responses on the ScanTron form and on the exam itself.*

**1.** Differentiate the function  $2 \cos x - \sin x$  with respect to  $x$ .

(a)  $-2 \sin x - \cos x$

(b)  $2 \cos x + \sin x$

(c)  $-2 \cos x + \sin x$

(d)  $-2 \sin x + \cos x$

(e)  $2 \cos x - \sin x$

**2.** Compute the slope of the tangent to the curve  $y = x - \sec x$  at the point  $(0, -1)$ .

(a)  $-1$

(b)  $1$

(c)  $-2$

(d)  $2$

(e)  $0$

**3.** Given that  $f(x) = (3x - 1)^8$ , find the value of  $f'(0)$ .

(a)  $-24$

(b)  $-16$

(c)  $-8$

(d)  $16$

(e)  $24$

4. Compute the 11th derivative of  $f(x) = \sin x$  with respect to  $x$ .
- (a)  $\sin x + \cos x$
  - (b)  $-\sin x$
  - (c)  $\sin x$
  - (d)  $\cos x$
  - (e)  $-\cos x$
5. Determine the tangent vector to the vector function  $\mathbf{r}(t) = \langle t^{1/3}, e^{2t} \rangle$ , corresponding to  $t = 1$ .
- (a)  $\langle 2e^2, 1/3 \rangle$
  - (b)  $\langle 1/3, e^2 \rangle$
  - (c)  $\langle e^2, 1/3 \rangle$
  - (d)  $\langle 1/3, 2e^2 \rangle$
  - (e)  $\langle 1, 1 \rangle$
6. Suppose that one begins to solve the equation  $x^3 - 2x - 5 = 0$  using Newton's method. If  $x_1$  is chosen to be 2, what is  $x_2$ ?
- (a)  $5/2$
  - (b)  $19/10$
  - (c)  $21/10$
  - (d)  $9/4$
  - (e)  $7/3$

7. Compute  $\lim_{x \rightarrow 0^-} \frac{1}{2 + 3^{(1/x)}}$ .
- (a)  $1/2$
  - (b)  $2$
  - (c)  $-2$
  - (d)  $-1/2$
  - (e)  $0$
8. Consider the functions  $f_1(x) = x^2$ ,  $f_2(x) = \cos(x)$ , and  $f_3(x) = \sin(x)$ . Which of these is **not** one-to-one on the interval  $[-\pi/2, \pi/2]$ ?
- (a)  $f_1$  and  $f_2$
  - (b)  $f_1$  and  $f_3$
  - (c)  $f_2$  and  $f_3$
  - (d)  $f_1$ ,  $f_2$ , and  $f_3$
  - (e) All three functions are one-to-one
9. Compute the linear approximation to  $f(x) = e^{\sin x}$  at  $x = \pi$ .
- (a)  $L(x) = \pi + 1 - x$
  - (b)  $L(x) = x + 1 - \pi$
  - (c)  $L(x) = 1 + e^{-1}(x - \pi)$
  - (d)  $L(x) = 1 + e(x - \pi)$
  - (e)  $L(x) = x - \pi$

10. The function  $f(x) = \frac{2}{1+x^2}$  can be shown to be one-to-one on the interval  $[0, \infty)$ . Determine  $f^{-1}$  (the inverse of  $f$ ), and state its domain.

(a)  $f^{-1}(x) = \sqrt{\frac{1-2x}{x}}$ ; domain= $(0, 1/2]$

(b)  $f^{-1}(x) = \frac{1-2x}{x}$ ; domain= $(0, \infty)$

(c)  $f^{-1}(x) = \sqrt{\frac{2-x}{x}}$ ; domain= $(0, 2]$

(d)  $f^{-1}(x) = \frac{2-x}{x}$ ; domain= $(0, \infty)$

(e)  $f^{-1}(x) = \frac{1+x^2}{2}$ ; domain= $(-\infty, \infty)$

11. Compute

$$\lim_{h \rightarrow 0} \frac{\tan^2\left(\frac{\pi}{4} + h\right) - 1}{h}.$$

(Hint: Recall the definition of a derivative at a point.)

(a)  $\infty$

(b) 2

(c) 0

(d)  $-\infty$

(e) 4

**Part 2 (61 points)**

*Present your solutions to the following problems (12–16) in the space provided. **Show all your work** neatly and concisely, and **indicate your final answer clearly**. You will be graded, not merely on the final answer, but also on the quality and correctness of the work leading up to it.*

**12. (12 points)** Verify that the function  $y = x^2e^x$  satisfies the following equation:

$$y''' - 3y'' + 3y' - y = 0.$$

13. (12 points) Given that

$$y\sqrt{x-1} + x\sqrt{y-1} = 3xy,$$

employ implicit differentiation to compute  $\frac{dy}{dx}$ .

14. A heap of rubbish in the shape of a cube is being compacted into a smaller cube in such a way that the volume decreases at the rate of 2 cubic meters per minute. (You may assume that the shape of the heap remains cubical at every instant.)

(i) (6 points) Find the rate of change of the length of an edge of the cube when the volume is 64 cubic meters.

(ii) (5 points) How fast is the surface area of the cube changing at that instant (*i.e.*, when the volume is 64 cubic meters)?

15. Let  $C$  denote the parametric curve determined by the equations

$$x(t) = \frac{1-t}{1+t}, \quad y(t) = \frac{1-\sqrt{t}}{1+t}, \quad t \geq 0.$$

(i) (7 points) Compute  $x'(t)$ .

(ii) (7 points) Compute  $y'(t)$ .

(iii) (6 points) Calculate the slope of the tangent to  $C$  at the point  $(0, 0)$ .

16. (6 points) It can be shown that the function

$$f(x) = e^{x^3+3x}, \quad -\infty < x < \infty,$$

is one-to-one. Let  $g$  denote the inverse of  $f$ . Find an equation of the tangent to the graph of  $y = g(x)$ , when  $x = 1$ .

**QN                      PTS**

1-11

12

13

14

15

16

**TOTAL**