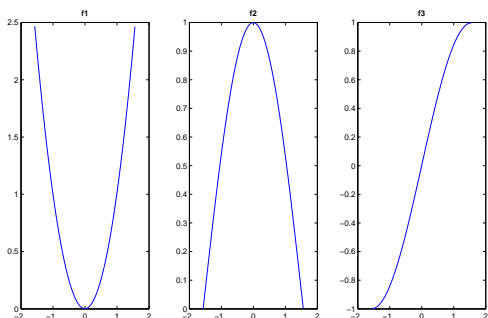


Exam II Solutions (Form B)

- A** $\frac{d}{dx}(2 \cos x - \sin x) = -2 \sin x - \cos x$
- B** $y' = 1 - \sec x \tan x$; when $x = 0$, $y' = 1 - \sec 0 \tan 0 = 1$
- A** $f'(x) = 8(3x - 1)^7(3)$ using the Chain Rule, so $f'(0) = 8(-1)^7(3) = -24$
- E** The derivatives of $\sin x$ follow the pattern $\cos x, -\sin x, -\cos x, \sin x, \dots$ so the eleventh derivative is the same as the third derivative, $-\cos x$.
- D** $\mathbf{r}'(t) = \langle 1/3 t^{-2/3}, 2e^{2t} \rangle$, so $\mathbf{r}'(1) = \langle 1/3, 2e^2 \rangle$
- C** Newton's Method says that

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 2 - \frac{2^3 - 2 \cdot 2 - 5}{3(2^2) - 2} = 2 - \frac{-1}{10} = 21/10$$
- A** Begin with the exponent: $\lim_{x \rightarrow 0^-} \frac{1}{x} = -\infty$. Thus $\lim_{x \rightarrow 0^-} 3^{1/x} = 0$ and $\lim_{x \rightarrow 0^-} \frac{1}{2 + 3^{(1/x)}} = \frac{1}{2 + 0} = \frac{1}{2}$
- A** The graphs of the functions are shown below. Using the horizontal line test, f_1 and f_2 are NOT one-to-one.



- A** Either use the formula $L(x) = f(a) + f'(a)(x - a)$ or just find the equation of the tangent line. $f(\pi) = e^{\sin \pi} = e^0 = 1$. $f'(x) = e^{\sin x} \cos x$, so $f'(\pi) = e^{\sin \pi} \cos \pi = -1$. The equation is $L(x) = 1 + (-1)(x - \pi) = \pi + 1 - x$.
- C** Switch x and y and solve for y : $x = \frac{2}{1 + y^2}$, $x + xy^2 = 2$, $xy^2 = 2 - x$, $y = \sqrt{\frac{2 - x}{x}}$ (we take the positive square root since the domain of the original function is $[0, \infty)$). The domain of this function is when the fraction is positive, which occurs when $x \in (0, 2]$.
- E** The limit is nothing more than the definition of $f'(a)$, where $f(x) = \tan^2 x$ and $a = \frac{\pi}{4}$.
 $f'(x) = 2 \tan x \sec^2 x$, so $f'(\pi/4) = 2 \tan\left(\frac{\pi}{4}\right) \sec^2\left(\frac{\pi}{4}\right) = 2(1)(\sqrt{2})^2 = 4$.
- Using the Product Rule, $y' = 2xe^x + x^2e^x = (2x + x^2)e^x$, $y'' = (2 + 2x)e^x + (2x + x^2)e^x = (2 + 4x + x^2)e^x$, $y''' = (4 + 2x)e^x + (2 + 4x + x^2)e^x = (6 + 6x + x^2)e^x$.
 Substituting all derivatives into the differential equation yields $(6 + 6x + x^2)e^x - 3(2 + 4x + x^2)e^x + 3(2x + x^2)e^x - x^2e^x = (6 + 6x + x^2 - 6 - 12x - 3x^2 + 6x + 3x^2 - x^2)e^x = 0$
- $y' \sqrt{x-1} + y(1/2)(x-1)^{-1/2} + \sqrt{y-1} + x(1/2)(y-1)^{-1/2}(y') = 3xy' + 3y$. Moving the terms with y' to the left and terms without y' to the right yields $y'(x-1)^{1/2} + (1/2)x(y-1)^{-1/2}y' - 3xy' = 3y - (1/2)y(x-1)^{-1/2} - (y-1)^{1/2}$, so $y' = \frac{3y - (1/2)y(x-1)^{-1/2} - (y-1)^{1/2}}{-3x + (1/2)x(y-1)^{-1/2} + (x-1)^{1/2}}$.

14. i) The relating formula is $V = x^3$, given $\frac{dV}{dt} = -2$. At the instant when $V = 64$, we know $x = 4$.
 Differentiate and substitute: $\frac{dV}{dt} = 3x^2 \frac{dx}{dt}$, $-2 = 3(4)^2 \frac{dx}{dt}$, or $\frac{dx}{dt} = -\frac{1}{24} \frac{m}{min}$.
- ii) The relating formula is $S = 6x^2$. Differentiate: $\frac{dS}{dt} = 12x \frac{dx}{dt}$. At the instant when $x = 4$, $\frac{dx}{dt} = -\frac{1}{24}$, so substitute: $\frac{dS}{dt} = 12x \frac{dx}{dt}$, $\frac{dS}{dt} = 12(4) \left(-\frac{1}{24}\right) = -2 \frac{m^2}{min}$.
15. i) Using the quotient rule $x'(t) = \frac{(1+t)(-1) - (1-t)(1)}{(1+t)^2} = \frac{-2}{(1+t)^2}$
- ii) Using the quotient rule $y'(t) = \frac{(1+t)(-1/2)t^{-1/2} - (1-t^{1/2})(1)}{(1+t)^2}$
- iii) The slope is found by $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$. When $x = 0$ and $y = 0$, we have $t = 1$, so the slope is
- $$\left. \frac{dy}{dx} \right|_{t=1} = \frac{\frac{(1+1)(-1/2)(1)^{-1/2} - (1-1^{1/2})(1)}{(1+1)^2}}{\frac{-2}{(1+1)^2}} = \frac{1}{2}$$
16. The slope of the tangent line is given by $g'(1) = \frac{1}{f'(g(1))}$. $f'(x) = e^{x^3+3x}(3x^2+3)$. To find $g(1)$, solve $f(x) = 1$. $e^{x^3+3x} = 1$ when $x^3+3x = x(x^2+3) = 0$, or $x = 0$ (Since $x^2+3x \neq 0$). So $g'(1) = \frac{1}{f'(0)} = \frac{1}{e^0(3 \cdot 0 + 3)} = \frac{1}{3}$. Since $g(1) = 0$, the tangent line passes through the point $(1, 0)$. Therefore, the equation is $y - 0 = \frac{1}{3}(x - 1)$, or $y = \frac{1}{3}x - \frac{1}{3}$.