MATH 151, SPRING SEMESTER 2009
COMMON EXAMINATION III - VERSION A

Name (print):_____

Signature: _____

Instructor's name: _____

Section No: _____

INSTRUCTIONS

1. In Part 1 (Problems 1–11), mark your responses on your ScanTron form using a No: 2 pencil. *For your own record, mark your choices on the exam as well.* Collected scantrons will *not* be returned after the examination.

2. Calculators **should not be used** throughout the examination.

3. In Part 2 (Problems 12–15), present your solutions in the space provided. **Show all your work** neatly and concisely, and **indicate your final answer clearly**. You will be graded, not merely on the final answer, but also on the quality and correctness of the work leading up to it.

4. Be sure to **write your name, section number, and version letter of the exam on the ScanTron form**.

Part 1 – Multiple Choice (44 points)

Each question is worth **4 points**. *Mark your responses on the ScanTron form and on the exam itself.*

1. If $\log_2(x+2) = 3$, what is x?

(a) 8

(b) 10

(c) 4

(d) 6

(e) 0

2. What is the domain of the function $f(x) = \cos^{-1}(x-2)$?

(a) $[-1, 1]$

(b) $[-3, -1]$

(c) $(-\infty, \infty)$

(d) $[2, \infty)$

(e) $[1, 3]$

3. Calculate the slope of the tangent to the curve $y = x - \tan^{-1}(x)$ at $x = 1$.

(a) 0

(b) $-1/2$

(c) $1/2$

(d) -1

(e) 1

4. Differentiate the function $f(x) = x^{\tan x}, \quad 0 < x < \pi/2.$

(a) $(\tan x)x^{\tan x - 1}$

(b) $x^{\sec x}\left(\dfrac{\sec x}{x} + \ln x \sec x \tan x\right)$

(c) $x^{\tan x}\left(\dfrac{\tan x}{x} + \ln x \sec^2 x\right)$

(d) $(\sec x)x^{\sec x - 1}$

(e) $x^{\tan x}\left(\dfrac{\sec^2 x}{x}\right)$

5. Compute $\lim\limits_{x \to 0^+} \dfrac{\ln x}{x^2}$.

(a) 1

(b) $+\infty$

(c) 0

(d) -1

(e) $-\infty$

6. Compute $\lim_{x \to 0} \dfrac{x \sin x}{\cos x - 1}$.

(a) 1

(b) 2

(c) 0

(d) -2

(e) does not exist

7. A bacteria culture starts with 500 bacteria and after 3 hours there are 8000 bacteria. Find an expression for the number of bacteria after t hours.

(a) $16^{t/3}$

(b) $(500)18^{t/3}$

(c) $(500)16^{t/3}$

(d) $18^{t/3}$

(e) $(500)16^{t}$

8. Which of the following values of x give rise to points of inflection on the graph of $y = \dfrac{x^2}{4} + \sin x, \quad 0 \leq x \leq \pi$.

(a) $x = \dfrac{\pi}{3}$ and $x = \dfrac{2\pi}{3}$

(b) $x = \dfrac{\pi}{6}$ and $x = \dfrac{5\pi}{6}$

(c) $x = \dfrac{\pi}{6}$ only

(d) $x = \dfrac{\pi}{3}$ only

(e) The graph has no points of inflection

9. Let t be a fixed number in the interval $(0, 1)$. Determine the value of $\tan(\sin^{-1}(t))$.

(a) $\dfrac{\sec^2(\sin^{-1}(t))}{\sqrt{1-t^2}}$

(b) $\dfrac{\sqrt{1-t^2}}{t}$

(c) t

(d) $\sqrt{1-t^2}$

(e) $\dfrac{t}{\sqrt{1-t^2}}$

10. Suppose that the function f is differentiable throughout $(-\infty, \infty)$, and that f has exactly one critical number, namely $x = 1$. Determine all the critical numbers of the function $h(x) = f(x^2)$.

(a) $x = 1/2$

(b) $x = 1$

(c) $x = -1, 1$

(d) $x = -1, 0, 1$

(e) cannot be determined with the information given

11. Suppose that g is differentiable on $(0, \infty)$, that $x^2 g'(x) = 2x^2 + 3x$ for every $x > 0$, and that $g(e) = 1$. Determine g.

(a) $g(x) = 3x + 2\ln x$

(b) $g(x) = 3x + 2\ln x - (1 + 3e)$

(c) $g(x) = 2x + 3\ln x$

(d) $g(x) = 2x + 3\ln x - (2 + 2e)$

(e) $g(x) = \dfrac{2}{3}x^3 + \dfrac{3}{2}x^2$

Part 2 (63 points)

Present your solutions to the following problems (12–15) in the space provided. **Show all your work** *neatly and concisely, and* **indicate your final answer clearly**. *You will be graded, not merely on the final answer, but also on the quality and correctness of the work leading up to it.*

12. Consider the function

$$H(x) = \ln\left(\sin^{-1}(x)\right).$$

(a) (`4 points`) Determine the domain of H.

(b) (`6 points`) Compute $H'(x)$.

(c) (`2 points`) Determine the domain of H'.

13. Consider the function

$$f(x) = xe^{-2x^2}, \qquad -\infty < x < \infty.$$

(a) (`7 points`) Verify that

$$f'(x) = (1 - 4x^2)e^{-2x^2} \quad \text{and} \quad f''(x) = -4x(3 - 4x^2)e^{-2x^2}, \qquad -\infty < x < \infty.$$

(b) (`4 points`) Find all the critical numbers of f.

(c) (`6 points`) Determine the intervals in which f is increasing, and those in which f is decreasing.

(d) (`2 points`) Find all the local maximum values of f.

(e) (`4 points`) Find the absolute maximum and absolute minimum values of f.

(f) (`4 points`) Determine the intervals in which f is concave up.

(g) (`4 points`) For how many values of x is the equation $f(x) = 1/4$ satisfied? Explain your reasoning clearly and concisely.

14. (`10 points`) Compute

$$\lim_{x\to\infty}\left(\frac{x-3}{x+4}\right)^{x}.$$

Show all your steps clearly and concisely.

15. (`10 points`) A paint manufacturer wants cylindrical cans for its specialty enamels. The can is to have a volume of 22 cubic inches. Determine the dimensions of the can that will require the least amount of material. Show that your answer does provide the stated minimum. (Note: The volume of a circular cylinder of radius r and height h is given by $\pi r^2 h$; its total surface area (including the top, bottom, and lateral areas) is $2\pi r^2 + 2\pi r h$.)

QN	PTS
1–11	
12	
13	
14	
15	
TOTAL	