

**MATH 151, SPRING SEMESTER 2009
COMMON EXAMINATION III - VERSION B**

Name (print): _____

Signature: _____

Instructor's name: _____

Section No: _____

INSTRUCTIONS

1. In Part 1 (Problems 1–11), mark your responses on your ScanTron form using a No: 2 pencil. *For your own record, mark your choices on the exam as well.* Collected scantrons will *not* be returned after the examination.
2. Calculators **should not be used** throughout the examination.
3. In Part 2 (Problems 12–15), present your solutions in the space provided. **Show all your work** neatly and concisely, and **indicate your final answer clearly**. You will be graded, not merely on the final answer, but also on the quality and correctness of the work leading up to it.
4. Be sure to **write your name, section number, and version letter of the exam on the ScanTron form.**

Part 1 – Multiple Choice (44 points)

Each question is worth 4 points. Mark your responses on the ScanTron form and on the exam itself.

1. If $\log_2(x - 2) = 3$, what is x ?

(a) 10

(b) 6

(c) 4

(d) 8

(e) 0

2. What is the domain of the function $f(x) = \cos^{-1}(x + 2)$?

(a) $[1, 3]$

(b) $[-3, -1]$

(c) $(-\infty, \infty)$

(d) $[2, \infty)$

(e) $[-1, 1]$

3. Calculate the slope of the tangent to the curve $y = \tan^{-1}(x) - x$ at $x = 1$.

(a) $1/2$

(b) -1

(c) 0

(d) $-1/2$

(e) 1

4. Differentiate the function $f(x) = x^{\sec x}$, $0 < x < \pi/2$.

(a) $x^{\tan x} \left(\frac{\tan x}{x} + \ln x \sec^2 x \right)$

(b) $x^{\sec x} \left(\frac{\sec x}{x} + \ln x \sec x \tan x \right)$

(c) $(\tan x)x^{\tan x-1}$

(d) $(\sec x)x^{\sec x-1}$

(e) $x^{\sec x} \left(\frac{\sec x \tan x}{x} \right)$

5. Compute $\lim_{x \rightarrow 0^+} \frac{\ln x}{x^2}$.

(a) $+\infty$

(b) $-\infty$

(c) 0

(d) -1

(e) 1

6. Compute $\lim_{x \rightarrow 0} \frac{x \sin x}{1 - \cos x}$.

(a) -2

(b) 0

(c) 2

(d) 1

(e) does not exist

7. A bacteria culture starts with 500 bacteria and after 3 hours there are 9000 bacteria. Find an expression for the number of bacteria after t hours.

(a) $(500)16^{t/3}$

(b) $18^{t/3}$

(c) $16^{t/3}$

(d) $(500)18^{t/3}$

(e) $(500)18^t$

8. Which of the following values of x give rise to points of inflection on the graph of

$$y = \frac{\sqrt{3}}{4}x^2 + \sin x, \quad 0 \leq x \leq \pi.$$

(a) $x = \frac{\pi}{6}$ and $x = \frac{5\pi}{6}$

(b) $x = \frac{\pi}{6}$ only

(c) $x = \frac{\pi}{3}$ and $x = \frac{2\pi}{3}$

(d) $x = \frac{\pi}{3}$ only

(e) The graph has no points of inflection

9. Let t be a fixed number in the interval $(0, 1)$. Determine the value of $\tan(\cos^{-1}(t))$.

(a) $\frac{t}{\sqrt{1-t^2}}$

(b) $\frac{\sqrt{1-t^2}}{t}$

(c) t

(d) $\sqrt{1-t^2}$

(e) $-\frac{\sec^2(\cos^{-1}(t))}{\sqrt{1-t^2}}$

10. Suppose that the function f is differentiable throughout $(-\infty, \infty)$, and that f has exactly one critical number, namely $x = 1$. Determine all the critical numbers of the function $h(x) = f(x^2)$.

(a) $x = -1, 0, 1$

(b) $x = 1$

(c) $x = -1, 1$

(d) $x = 1/2$

(e) cannot be determined with the information given

11. Suppose that g is differentiable on $(0, \infty)$, that $x^2g'(x) = 3x^2 + 2x$ for every $x > 0$, and that $g(e) = 1$. Determine g .

(a) $g(x) = 2x + 3 \ln x - (2 + 2e)$

(b) $g(x) = 3x + 2 \ln x - (1 + 3e)$

(c) $g(x) = 2x + 3 \ln x$

(d) $g(x) = 3x + 2 \ln x$

(e) $g(x) = x^3 + x^2$

Part 2 (63 points)

Present your solutions to the following problems (12–15) in the space provided. Show all your work neatly and concisely, and indicate your final answer clearly. You will be graded, not merely on the final answer, but also on the quality and correctness of the work leading up to it.

12. Consider the function

$$H(x) = \ln(\sin^{-1}(x)).$$

(a) (4 points) Determine the domain of H .

(b) (6 points) Compute $H'(x)$.

(c) (2 points) Determine the domain of H' .

13. Consider the function

$$f(x) = xe^{-2x^2}, \quad -\infty < x < \infty.$$

(a) (7 points) Verify that

$$f'(x) = (1 - 4x^2)e^{-2x^2} \quad \text{and} \quad f''(x) = -4x(3 - 4x^2)e^{-2x^2}, \quad -\infty < x < \infty.$$

(b) (4 points) Find all the critical numbers of f .

(c) (6 points) Determine the intervals in which f is increasing, and those in which f is decreasing.

(d) (2 points) Find all the local minimum values of f .

(e) (4 points) Find the absolute maximum and absolute minimum values of f .

(f) (4 points) Determine the intervals in which f is concave down.

(g) (4 points) For how many values of x is the equation $f(x) = 1/4$ satisfied? Explain your reasoning clearly and concisely.

14. (10 points) Compute

$$\lim_{x \rightarrow \infty} \left(\frac{x-3}{x+5} \right)^x .$$

Show all your steps clearly and concisely.

- 15. (10 points)** A paint manufacturer wants cylindrical cans for its specialty enamels. The can is to have a volume of 24 cubic inches. Determine the dimensions of the can that will require the least amount of material. Show that your answer does provide the stated minimum. (Note: The volume of a circular cylinder of radius r and height h is given by $\pi r^2 h$; its total surface area (including the top, bottom, and lateral areas) is $2\pi r^2 + 2\pi r h$.)

QN PTS

1-11 _____

12 _____

13 _____

14 _____

15 _____

TOTAL