Math 151 Fall 2009 Exam I Solutions-Form B

1. D: The vector $\langle 3,2\rangle$ is parallel to the line $x=3t+1$, $y=2t+5$. Hence $\langle 2,-3\rangle$ is perpendicular to the line. Divide by the magnitude to make the vector a unit vector. $\dfrac{\langle 2,-3\rangle}{\sqrt{13}}=\left\langle \dfrac{2}{\sqrt{13}},-\dfrac{3}{\sqrt{13}}\right\rangle$.

2. A: $\displaystyle\lim_{x\to 0}\frac{\sqrt{9+x}-3}{x}=\lim_{x\to 0}\frac{\sqrt{9+x}-3}{x}\frac{\sqrt{9+x}+3}{\sqrt{9+x}+3}$
$\displaystyle=\lim_{x\to 0}\frac{9+x-9}{x(\sqrt{9+x}+3)}=\frac{1}{6}$.

3. C: Use the product rule to differentiate $H(x)=x^2f(x)$: $H'(x)=x^2f'(x)+2xf(x)$. Since $f(2)=4$ and $f'(2)=-3$,
$H'(2)=(2)^2f'(2)+2(2)f(2)$
$=4(-3)+(4)(4)=4$

4. A: $W=|\mathbf{F}||\mathbf{D}|\cos(\theta)$. Here, $|\mathbf{F}|=8$ pounds, $|\mathbf{D}|=25$ feet and $\theta=30°$. Hence
$W=(8 \text{ pounds})\ (25 \text{ feet})\ (\cos 30°)$
$=100\sqrt{3}$ foot pounds.

5. B: Use the quotient rule to differentate $f(x)=\dfrac{x}{1+x}$.
$f'(x)=\dfrac{(1)(1+x)-(x)(1)}{(1+x)^2}=\dfrac{1}{(1+x)^2}$. Hence the slope of the tangent line is $m=f'(2)=\dfrac{1}{9}$. Also, when $x=2$, $y=\dfrac{2}{3}$. Thus the equation of the tangent line is $y-\dfrac{2}{3}=\dfrac{1}{9}(x-2)$.

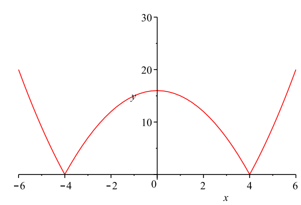
6. B: $x=4+\sin t$ and $y=\cos t$ is equivalent to $x-4=\sin t$, $y=\cos t$. Since $\sin^2 t+\cos^2 t=1$, $(x-4)^2+y^2=1$.

7. A: To test $f(x)=\begin{cases}2x+1 & \text{if } x\le -1\\ x^2-2 & \text{if } -1<x\le 2\\ \dfrac{1}{x}+\dfrac{1}{2} & \text{if } x>2\end{cases}$ for continuity, we first note that $2x+1$ is continuous for $x\le -1$, x^2-2 is continuous for $-1<x\le 2$ and $\dfrac{1}{x}+\dfrac{1}{2}$ is continuous for $x>2$. Thus we need only to check continuity for $x=-1$ and $x=2$. $\displaystyle\lim_{x\to 2^-}f(x)=2$, $\displaystyle\lim_{x\to 2^+}f(x)=1$.
Thus $\displaystyle\lim_{x\to 2}f(x)$ does not exist, hence $f(x)$ is not continuous at $x=2$.
$\displaystyle\lim_{x\to -1^-}f(x)=-1$, $\displaystyle\lim_{x\to -1^+}f(x)=-1$, $f(-1)=-1$, thus $f(x)$ is continuous at $x=-1$. The only place of discontinuity is $x=2$.

8. C: $\displaystyle\lim_{x\to -3^-}\frac{x-1}{x^2(x+3)}=\infty$ since $x=-3$ is a vertical asymptote and $\dfrac{x-1}{x^2(x+3)}>0$ for $x<-3$.

9. E: First, note $f(x)=x^3+x^2-3x-10$ is continuous, so we will apply the Intermediate Value Theorem. Since $f(2)=-4<0$ and $f(3)=17>0$, there is a solution to $f(c)=0$ on the interval $(2,3)$.

10. C: $f(x)=|x^2-16|$ is not differentiable at $x=\pm 4$ since $'f(4)$ and $f'(-4)$ does not exist. (See figure below)



11. Multiply by the conjugate:
$$\lim_{x\to -\infty}\left(x+\sqrt{x^2+5x}\right)\frac{x-\sqrt{x^2+5x}}{x-\sqrt{x^2+5x}}$$
$$=\lim_{x\to\infty}\frac{-5x}{x-\sqrt{x^2+5x}}$$
$$=\lim_{x\to\infty}\frac{-5x}{x-|x|}$$
$$=\lim_{x\to\infty}\frac{-5x}{x-(-x)}$$
$$=\lim_{x\to\infty}\frac{-5x}{2x}=-\frac{5}{2}$$

12. (i) To find the cosine of the angle between the vectors $\langle 1,2\rangle$ and $\langle 3,4\rangle$, we will use the formula

$$\cos\theta = \frac{\langle 1,2\rangle \cdot \langle 3,4\rangle}{(|\langle 1,2\rangle|)(|\langle 3,4\rangle|)}$$

$$\cos\theta = \frac{3+8}{\sqrt{5}\sqrt{25}} = \frac{11}{5\sqrt{5}}.$$

(ii) Let $\mathbf{a} = \langle 3,4\rangle$ and $\mathbf{b} = \langle 1,2\rangle$ Then

$$\text{comp}_{\mathbf{a}}\mathbf{b} = \frac{\langle 3,4\rangle \cdot \langle 1,2\rangle}{|\langle 3,4\rangle|} = \frac{11}{5}$$

(iii) $\text{proj}_{\mathbf{a}}\mathbf{b} = \dfrac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}|^2}\mathbf{a}$

$$= \frac{\langle 3,4\rangle \cdot \langle 1,2\rangle}{|\langle 3,4\rangle|^2}\langle 3,4\rangle$$

$$= \left\langle \frac{33}{25}, \frac{44}{25}\right\rangle$$

13. $f(x) = \dfrac{3}{x-2}$. $f'(x) = \lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$

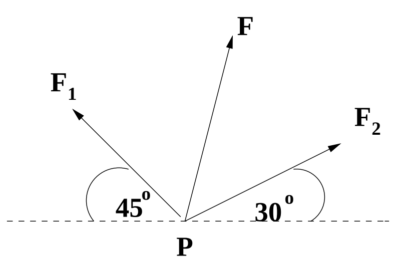
$$= \lim_{h\to 0}\frac{\frac{3}{x+h-2} - \frac{3}{x-2}}{h}$$

$$= \lim_{h\to 0}\frac{3(x-2)-3(x+h-2)}{h(x+h-2)(x-2)}$$

$$= \lim_{h\to 0}\frac{-3h}{h(x+h-2)(x-2)}$$

$$= \frac{-3}{(x-2)^2}$$

14.



Since $|\mathbf{F_1}| = 10$ pounds and $|\mathbf{F_2}| = 8$ pounds,

$\mathbf{F_1} = \langle -10\cos 45°, 10\sin 45°\rangle = \left\langle -5\sqrt{2}, 5\sqrt{2}\right\rangle$.

$\mathbf{F_2} = \langle 8\cos 30°, 8\sin 30°\rangle = \left\langle 4\sqrt{3}, 4\right\rangle$. Thus the resultant force is

$\mathbf{F} = \mathbf{F_1} + \mathbf{F_2} = \left\langle -5\sqrt{2}+4\sqrt{3}, 5\sqrt{2}+4\right\rangle$. Thus the magnitude of the resultant force is

$$|\mathbf{F}| = \sqrt{\left(-5\sqrt{2}+4\sqrt{3}\right)^2 + \left(5\sqrt{2}+4\right)^2} \text{ pounds}$$

15. a.) Since $x < 2$, $|x-2| = -(x-2)$. Thus $\lim\limits_{x\to 2^-}\dfrac{x-2}{|x-2|} = \lim\limits_{x\to 2^-}\dfrac{x-2}{-(x-2)}$
$= -1$

b.) To find the value of a for which $\lim\limits_{x\to 1}\dfrac{3x^2+ax+a+5}{x^2+x-2}$ exists, we use the fact that, in order for the limit to exist at $x=1$, $3x^2+ax+a+5$ must have a factor of $x-1$ in order to eliminate the division by zero. Hence if $x-1$ is a factor of $3x^2+ax+a+5$, it follows that

$3(1)^2 + a(1) + a + 5 = 0$. Thus $3+a+a+5=0$ yielding $a=-4$.