Math 151 Fall 2009 Exam III Solutions-Form A

1. C: $\lim_{x\to 0} \frac{x-\arcsin(2x)}{x+\arctan x} = \frac{0}{0}$. Thus by L'Hospital's rule,

$$\lim_{x\to 0} \frac{x-\arcsin(2x)}{x+\arctan x} = \lim_{x\to 0} \frac{1-\frac{2}{\sqrt{1-4x^2}}}{1+\frac{1}{1+x^2}} = -\frac{1}{2}$$

2. A: $f(x) = x\ln x + 2x$. Thus $f'(x) = \ln x + x\frac{1}{x} + 2$ $= \ln x + 3$. Thus $f'(x) = 0$ if $\ln x + 3 = 0$, hence $\ln x = -3$. Solving for x yields $x = e^{-3} = \frac{1}{e^3}$.

3. C: $f'(x) = 3\cos x - 5\sin x$. Take the antiderivative of $f'(x)$ to get $f(x)$. $f(x) = 3\sin x + 5\cos x + C$. We are given $f(0) = 6$, thus $6 = 3\sin(0) + 5\cos(0) + C$, hence $C = 1$. $f(x) = 3\sin x + 5\cos x + 1$ and therefore $f(\pi) = 3\sin(\pi) + 5\cos(\pi) + 1 = -4$.

4. E: $\arctan\left(\tan\frac{2\pi}{3}\right) = \arctan\left(-\sqrt{3}\right) = -\frac{\pi}{3}$

5. E: $f(x)$ is concave down where $f'(x)$ is decreasing. This occurs on (b, d).

6. C: $y = x^{\sin x}$, thus $\ln y = \ln x^{\sin x} = \sin x \ln x$. Differentiate implicitly with respect to x:
$\frac{1}{y}\frac{dy}{dx} = \cos x \ln x + \frac{\sin x}{x}$,
hence $\frac{dy}{dx} = y\left(\cos x \ln x + \frac{\sin x}{x}\right)$, therefore
$\frac{dy}{dx} = x^{\sin x}\left(\cos x \ln x + \frac{\sin x}{x}\right)$

7. C: $\sum_{i=1}^{4} a_i = 1$ and $\sum_{i=1}^{4} b_i = -2$, find $\sum_{i=1}^{4}(a_i + 2b_i + 2)$.
$\sum_{i=1}^{4}(a_i + 2b_i + 2) = \sum_{i=1}^{4} a_i + 2\sum_{i=1}^{4} b_i + \sum_{i=1}^{4} 2$
$= 1 + 2(-2) + 8 = 5$.

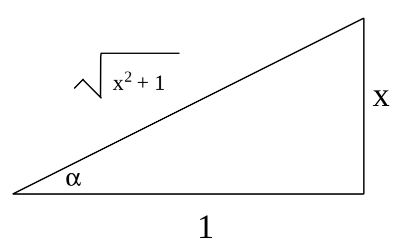
8. C: To find the absolute maximum for
$f(x) = x^3 - 12x + 1$ on the interval $[1, 3]$, we will first find the critical numbers of $f(x)$ for $0 \le x \le 3$. $f'(x) = 3x^2 - 12 = 3(x^2 - 4)$. Thus the only critical number for $f(x)$ on the interval $[1, 3]$ is $x = 2$. Now, $f(1) = -10$, $f(3) = -8$ and $f(2) = -15$. Hence the absolute maximum is -8.

9. B: $f(x) = 2\ln(\arctan x)$, thus $f'(x) = \frac{\frac{2}{1+x^2}}{\arctan x}$, hence $f'(1) = \frac{1}{\frac{\pi}{4}} = \frac{4}{\pi}$.

10. C: To find where $f(x) = xe^{3x}$ is decreasing, we will solve $f'(x) < 0$. Now by the product rule,
$f'(x) = e^{3x} + 3xe^{3x} = e^{3x}(1 + 3x)$. The only critical number for $f(x)$ is $x = -\frac{1}{3}$. Now $f'(x) < 0$ for $x < -\frac{1}{3}$ and $f'(x) > 0$ for $x > -\frac{1}{3}$. Hence $f(x)$ is decreasing on the interval $(-\infty, -\frac{1}{3})$.

11. C: To find the inflection points for $f(x) = x^4 - 6x^2$, we will determine where $f(x)$ changes concavity by looking at the sign of $f''(x)$. $f'(x) = 4x^3 - 6x^2$ and hence $f''(x) = 12x^2 - 12$. Nnow $f''(x) = 0$ when $x = \pm 1$. $f''(x) < 0$ for $-1 < x < 1$ and $f''(x) > 0$ for $x < -1$ and $x > 1$. Thus $f(x)$ goes from concave up to concave down at $x = -1$ and $f(x)$ goes from concave down to concave up at $x = 1$. Thus both $x = 1$ and $x = -1$ yield inflection points for $f(x)$.

12. A: Let $\alpha = \arctan x$. Then $\tan\alpha = x$. By viewing the triangle, we see that $\cos(\alpha) = \frac{1}{\sqrt{1+x^2}}$.



13. To solve for x: $\log_4(x^2-16)-\log_4(1-2x)=1$, we will use logarithim properties:

$\log_4(x^2-16)-\log_4(1-2x)=1$ is equivalent to $\log_4\dfrac{x^2-16}{1-2x}=1$. Thus $\dfrac{x^2-16}{1-2x}=4$. Cross multiply:

$x^2-16=4(1-2x)$ thus $x^2+8x-20=0$. Factoring gives $(x+10)(x-2)=0$, hence $x=-10$ or $x=2$. Now $x=2$ is not in the domain of $f(x)$ since we cannot take the logarithm of a negative number. Thus the only solution is $x=-10$.

14. Note $\lim\limits_{x\to\infty}(e^{2x}+x)^{1/x}$ is of the form ∞^0 which is an indeterminate power. Let $y=(e^{2x}+x)^{1/x}$. Then $\ln y=\ln(e^{2x}+x)^{1/x}=\dfrac{\ln(e^{2x}+x)}{x}$.

Now, $\lim\limits_{x\to 0}\dfrac{\ln(e^{2x}+x)}{x}$ is of the form $\dfrac{0}{0}$, so we can apply L'Hospital's rule.

$\lim\limits_{x\to 0}\dfrac{\ln(e^{2x}+x)}{x}=\lim\limits_{x\to 0}\dfrac{2e^{2x}+1}{e^{2x}+x}=3$. Thus $\lim\limits_{x\to 0}(e^{2x}+x)^{1/x}=e^3$

15. Using the formula $y(t)=(y_0-T)e^{kt}+T$ where $y(t)$ is the temperature of the object at time t, $y_0=375$ and $T=75$, we obtain $y(t)=300e^{kt}+75$. Now we are given that $y(20)=200$, thus

$200=300e^{k(20)}+75\Rightarrow 125=300e^{k(20)}\Rightarrow$

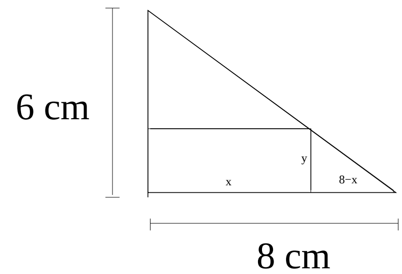
$\dfrac{5}{12}=e^{20k}$, thus $k=\dfrac{1}{20}\ln\dfrac{5}{12}$. Substitute

$k=\dfrac{1}{20}\ln\dfrac{5}{12}$ into $y(t)=300e^{kt}+75$, we find

$y(t)=300e^{(1/20)\ln(5/12)t}+75\Rightarrow$

$y(t)=300\left(\dfrac{5}{12}\right)^{t/20}+75.$

16. Let x and y be as the figure shows.



We want to maximize the the area of the rectangle $A=xy$. Now by similar triangles, $\dfrac{y}{8-x}=\dfrac{6}{8}$. Thus $y=\dfrac{3}{4}(8-x)$. Substitute this in for y gives us $A=x\dfrac{3}{4}(8-x)=6x-\dfrac{3}{4}x^2$. $A'=6-\dfrac{3}{2}x$. Solving $A'=0$ yields $x=4$. Now to determine whether this maximizes area we will apply the second derivative test: $A''=-\dfrac{3}{2}<0$, meaning A is concave down thus $x=4$ does indeed produce a maximum. Therefore the maximum area is $A=6(4)-\dfrac{3}{4}(4)^2=12$ square centimeters.

17. First partition the interval $[-1,7]$ into 4 subintervals of equal width. $\Delta x=\dfrac{7-(-1)}{4}=2$. Thus the partition is $\{-1,1,3,5,7\}$. Since we are using right endpoints, $x_1=1$, $x_2=3$, $x_3=5$ and $x_4=7$.

$\sum\limits_{i=1}^{4}f(x_i)\Delta x=(f(1)+f(3)+f(5)+f(7))\,(2)$

$=(3+11+27+51)\,(2)=184$