Spring 2010 Math 151

Exam I Version A Solutions

1. **C** $\lim_{x\to 2} \frac{x^2+x-6}{x^2-5x+6} = \lim_{x\to 2} \frac{(x-2)(x+3)}{(x-2)(x-3)} = \frac{2+3}{2-3} = -5.$

2. **D** $\lim_{x\to 5^-} f(x) = 4 - \frac{3}{5}(5) = 1$, $\lim_{x\to 5^+} f(x) = -1+5 = 4$, and $f(5) = 4$. Since $\lim_{x\to 5^+} f(x) = f(5)$ and $\lim_{x\to 5^-} f(x) \neq f(5)$, f is continuous only from the right.

3. **D** $\lim_{x\to 3} -\frac{1}{3}x + 2 = -\frac{1}{3}(3) + 2 = 1$ and $\lim_{x\to 3} \frac{3}{x} = \frac{3}{3} = 1$, therefore, by the Squeeze Theorem, $\lim_{x\to 3} f(x) = 1$.

4. **A** $\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{x^2\left(1-\frac{4}{x^2}\right)}{x^2\left(\frac{1}{x^2}-9\right)} = -\frac{1}{9}.$

   Similarly, $\lim_{x\to -\infty} f(x) = -\frac{1}{9}$, therefore, f has a horizontal asymptote at $y = -\frac{1}{9}$

5. **B** f has vertical asymptotes when the denominator approaches 0 and the numerator does not. $1-9x^2 = 0$ when $x = \pm\frac{1}{3}$. Since the numerator is not zero at these values, f has vertical asymptotes at $x = \frac{1}{3}, -\frac{1}{3}$.

6. **C** Let $f(x) = x^5 + x^2 - 2x$. $f(0) = 0$, $f(1) = 0$, and $f(2) = 32$. Since f is a polynomial, it is continuous, and since $0 < 3 < 32$, there is a solution to $f(x) = 3$ on the interval $[1, 2]$ by the Intermediate Value Theorem.

7. **B** $2\mathbf{a} + \mathbf{b} = (2\cdot(-4)+3)\mathbf{i} + (2\cdot 1 + 5)\mathbf{j} = -5\mathbf{i} + 7\mathbf{j}$. Form a unit vector $\mathbf{u}$ by multiplying this vector by the reciprocal of its magnitude, $\sqrt{(-5)^2+7^2} = \sqrt{74}$. Therefore, $\mathbf{u} = \frac{-5}{\sqrt{74}}\mathbf{i} + \frac{7}{\sqrt{74}}\mathbf{j}$.

8. **A** Let $\mathbf{a}$ be the vector from A to B. Then $\mathbf{a} = (1-(-2))\mathbf{i} + (3-4)\mathbf{j} = 3\mathbf{i} - \mathbf{j}$. Let $\mathbf{b}$ be the vector from A to C. Then $\mathbf{b} = (2-(-2))\mathbf{i} + (5-4)\mathbf{j} = 4\mathbf{i} + \mathbf{j}$. Then $\cos A = \frac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}||\mathbf{b}|} = \frac{(3)(4)+(-1)(1)}{\sqrt{3^2+(-1)^2}\sqrt{4^2+1^2}} = \frac{11}{\sqrt{170}}$.

9. **E** As $x \to -3^+$, the numerator approaches -3 and the denominator approaches 0 but is positive (since $x > -3$). Therefore, $\lim_{x\to -3^+} \frac{x}{x+3} = -\infty$.

10. **D** The derivative (slope of the tangent line) is zero when $x = 0$ and approaches zero when $x \to \pm\infty$. When $x < 0$, the derivative is negative, and when $x > 0$, the derivative is positive. Graph (d) is the only graph which satisfies these conditions.

11. **D** The line has vector equation $\mathbf{r}(t) = \mathbf{r_0} + t\mathbf{v}$. $\mathbf{r_0} =< -1, 1 >$ since the line passes through the point $(-1, 1)$. $\mathbf{v} =< 2, 3 > - < -1, 1 >=< 3, 2 >$. Therefore, the equation of the line is $\mathbf{r}(t) =< -1, 1 > +t < 3, 2 >=< -1+3t, 1+2t >$.

12. **B** The slope of the tangent line is $\lim_{h\to 0} \frac{f(2+h)-f(2)}{h} = \lim_{h\to 0} 3 + 5h + h^2 = 3$. Since the line is tangent to f at $(2, -4)$, the equation of the line is $y - (-4) = 3(x-2)$, or $y = 3x - 10$.

13. $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\frac{2}{3+x+h} - \frac{2}{3+x}}{h} = \lim_{h\to 0} \frac{1}{h}\left(\frac{2(3+x)-2(3+x+h)}{(3+x+h)(3+x)}\right) = \lim_{h\to 0} \frac{1}{h}\cdot\frac{-2h}{(3+x+h)(3+x)} = \lim_{h\to 0} \frac{-2}{(3+x+h)(3+x)} = \frac{-2}{(3+x)^2}$.

14. .

    (a) For each point, set $1+4t = x$-coordinate and $2-2t = y$-coordinate. Line ℓ passes through $(1, 2)$ when $t = 0$ and passes through $(5, 0)$ when $t = 1$.

    (b) Let Q be either point above (the point $(5, 0)$ shown here). Then $\mathbf{b} =< -1, 2 > - < 5, 0 >=< -6, 2 >$

    (c) Using the coefficients of t, $\mathbf{v} =< 4, -2 >$ is parallel to ℓ, so $\mathbf{a} = \mathbf{v}^\perp =< 2, 4 >$ is orthogonal to ℓ.

(d) The distance from P to ℓ is simply $|\text{comp}_{\mathbf{a}}\mathbf{b}| = \left|\dfrac{(-6)(2)+(2)(4)}{\sqrt{2^2+4^2}}\right| = \dfrac{4}{\sqrt{20}} = \dfrac{2}{\sqrt{5}}$.

15. Multiply by the conjugate: $\lim_{x\to\infty} \dfrac{1(\sqrt{x^2-3x+2}+x)}{(\sqrt{x^2-3x+2}-x)(\sqrt{x^2-3x+2}+x)}$ $= \lim_{x\to\infty} \dfrac{\sqrt{x^2-3x+2}+x}{x^2-3x+2-x^2}$. Since $x>0$, $\sqrt{x^2}=|x|=x$. $= \lim_{x\to\infty} \dfrac{x\left(\sqrt{1-\dfrac{3}{x}+\dfrac{2}{x^2}}+1\right)}{x\left(-3+\dfrac{2}{x}\right)} = \dfrac{\sqrt{1+0+0}+1}{-3+0} = -\dfrac{2}{3}$

16. The velocity vector for the woman is $\mathbf{w} =< -4, 0 >$. The velocity vector for the ship is $\mathbf{s} =< 20\cos 120°, 20\sin 120° >=< -10, 10\sqrt{3} >$. The velocity of the woman relative to the water is the resultant: $\mathbf{w}+\mathbf{s} =< -14, 10\sqrt{3} >$. Therefore, the speed of the woman relative to the water is $|\mathbf{w}+\mathbf{s}| = \sqrt{(-14)^2+(10\sqrt{3})^2} = \sqrt{496}$mph.

17. $\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-} cx^2+4x = 9c+12$. $\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} x^3+cx+1 = 27+3c+1$. For the limit to exist, the one-sided limits must be equal: $9c+12 = 3c+28$, $6c = 16$, $c = \dfrac{8}{3}$. For f to be continuous, the limit must equal $f(3) = K$, so set $K =$ either one-sided limit: $K = \dfrac{8}{3}(3)^2+4(3) = 36$.