Spring 2010 Math 151

Exam I Version B Solutions

1. **A** $\lim_{x\to 2} \frac{x^2-5x+6}{x^2+x-6} = \lim_{x\to 2} \frac{(x-2)(x-3)}{(x-2)(x+3)} = \frac{2-3}{2+3} = -\frac{1}{5}$.

2. **B** $\lim_{x\to 5^-} f(x) = 4 - \frac{3}{5}(5) = 1$, $\lim_{x\to 5^+} f(x) = -1+5 = 4$, and $f(5) = 1$. Since $\lim_{x\to 5^-} f(x) = f(5)$ and $\lim_{x\to 5^+} f(x) \neq f(5)$, f is continuous only from the left.

3. **B** $\lim_{x\to 2} - x + 4 = -2 + 4 = 2$ and $\lim_{x\to 2} \frac{4}{x} = \frac{4}{2} = 2$, therefore, by the Squeeze Theorem, $\lim_{x\to 2} f(x) = 2$.

4. **D** $\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{x^2\left(1-\frac{9}{x^2}\right)}{x^2\left(\frac{1}{x^2}-4\right)} = -\frac{1}{4}$.

   Similarly, $\lim_{x\to -\infty} f(x) = -\frac{1}{4}$, therefore, f has a horizontal asymptote at $y = -\frac{1}{4}$

5. **E** f has vertical asymptotes when the denominator approaches 0 and the numerator does not. $1-4x^2 = 0$ when $x = \pm\frac{1}{2}$. Since the numerator is not zero at these values, f has vertical asymptotes at $x = \frac{1}{2}, -\frac{1}{2}$.

6. **D** Let $f(x) = x^5 + x^2 + 2x$. $f(0) = 0$, $f(1) = 4$, and $f(2) = 40$. Since f is a polynomial, it is continuous, and since $0 < 3 < 4$, there is a solution to $f(x) = 3$ on the interval $[0, 1]$ by the Intermediate Value Theorem.

7. **E** $\mathbf{a} + 2\mathbf{b} = (-4+2\cdot 3)\mathbf{i} + (1+2\cdot 5)\mathbf{j} = 2\mathbf{i} + 11\mathbf{j}$. Form a unit vector $\mathbf{u}$ by multiplying this vector by the reciprocal of its magnitude, $\sqrt{2^2+11^2} = \sqrt{125}$. Therefore, $\mathbf{u} = \frac{2}{\sqrt{125}}\mathbf{i} + \frac{11}{\sqrt{125}}\mathbf{j}$.

8. **B** Let $\mathbf{a}$ be the vector from C to B. Then $\mathbf{a} = (1-2)\mathbf{i} + (3-5)\mathbf{j} = -\mathbf{i} - 2\mathbf{j}$. Let $\mathbf{b}$ be the vector from C to A. Then $\mathbf{b} = ((-2)-2)\mathbf{i} + (4-5)\mathbf{j} = -4\mathbf{i} - \mathbf{j}$. Then $\cos A = \frac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}||\mathbf{b}|} = \frac{(-1)(-4)+(-2)(-1)}{\sqrt{(-1)^2+(-2)^2}\sqrt{(-4)^2+(-1)^2}} = \frac{6}{\sqrt{85}}$.

9. **C** As $x \to -3^-$, the numerator approaches -3 and the denominator approaches 0 but is negative (since $x < -3$). Therefore, $\lim_{x\to -3^+} \frac{x}{x+3} = \infty$.

10. **E** The derivative (slope of the tangent line) is zero when $x = 0$ and approaches zero when $x \to \pm\infty$. When $x < 0$, the derivative is positive, and when $x > 0$, the derivative is negative. Graph (e) is the only graph which satisfies these conditions.

11. **C** The line has vector equation $\mathbf{r}(t) = \mathbf{r_0} + t\mathbf{v}$. $\mathbf{r_0} =< -1, 1 >$ since the line passes through the point $(-1, 1)$. $\mathbf{v} =< 1, 4 > - < -1, 1 >=< 2, 3 >$. Therefore, the equation of the line is $\mathbf{r}(t) =< -1, 1 > +t < 2, 3 >=< -1+2t,\ 1+3t >$.

12. **A** The slope of the tangent line is $\lim_{h\to 0} \frac{f(2+h)-f(2)}{h} = \lim_{h\to 0} 5 + 3h + h^2 = 5$. Since the line is tangent to f at $(2, -4)$, the equation of the line is $y - (-4) = 5(x-2)$, or $y = 5x - 14$.

13. $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\frac{3}{2+x+h} - \frac{3}{2+x}}{h} = \lim_{h\to 0} \frac{1}{h}\left(\frac{3(2+x)-3(2+x+h)}{(2+x+h)(2+x)}\right) = \lim_{h\to 0} \frac{1}{h}\cdot\frac{-3h}{(2+x+h)(2+x)} = \lim_{h\to 0} \frac{-3}{(2+x+h)(2+x)} = \frac{-3}{(2+x)^2}$.

14. .

    (a) For each point, set $1+4t = x$-coordinate and $2-2t = y$-coordinate. Line ℓ passes through $(-1, 2)$ when $t = 0$ and passes through $(-2, 0)$ when $t = -1$.

    (b) Let Q be either point above (the point $(-2, 0)$ shown here). Then $\mathbf{b} =< 4, -2 > - < -2, 0 >=< 6, -2 >$

    (c) Using the coefficients of t, $\mathbf{v} =< 1, 2 >$ is parallel to ℓ, so $\mathbf{a} = \mathbf{v}^{\perp} =< -2, 1 >$ is orthogonal to ℓ.

(d) The distance from P to ℓ is simply $|\text{comp}_{\mathbf{a}}\mathbf{b}| = \left|\dfrac{(6)(-2)+(-2)(1)}{\sqrt{(-2)^2+1^2}}\right| = \dfrac{14}{\sqrt{5}}$.

15. Multiply by the conjugate: $\lim\limits_{x\to\infty} \dfrac{1(\sqrt{x^2-5x+2}+x)}{(\sqrt{x^2-5x+2}-x)(\sqrt{x^2-5x+2}+x)}$ $= \lim\limits_{x\to\infty} \dfrac{\sqrt{x^2-5x+2}+x}{x^2-5x+2-x^2}$. Since $x > 0$, $\sqrt{x^2} = |x| = x$.
$= \lim\limits_{x\to\infty} \dfrac{x\left(\sqrt{1-\dfrac{5}{x}+\dfrac{2}{x^2}}+1\right)}{x\left(-5+\dfrac{2}{x}\right)} = \dfrac{\sqrt{1+0+0}+1}{-5+0} = -\dfrac{2}{5}$

16. The velocity vector for the woman is $\mathbf{w} =< -4, 0 >$. The velocity vector for the ship is $\mathbf{s} =< 20\cos 120°, 20\sin 120° >=< -10, 10\sqrt{3} >$. The velocity of the woman relative to the water is the resultant: $\mathbf{w}+\mathbf{s} =< -14, 10\sqrt{3} >$. Therefore, the speed of the woman relative to the water is $|\mathbf{w}+\mathbf{s}| = \sqrt{(-14)^2+(10\sqrt{3})^2} = \sqrt{496}$mph.

17. $\lim\limits_{x\to 3^-} f(x) = \lim\limits_{x\to 3^-} cx^2 - x = 9c - 3$.
$\lim\limits_{x\to 3^+} f(x) = \lim\limits_{x\to 3^+} x^3 - cx - 2 = 27 - 3c - 2$.
For the limit to exist, the one-sided limits must be equal: $9c - 3 = -3c + 25$, $12c = 28$, $c = \dfrac{7}{3}$. For f to be continuous, the limit must equal $f(3) = K$, so set $K =$ either one-sided limit: $K = \dfrac{7}{3}(3)^2 - (3) = 18$.