Spring 2010 Math 151

Exam II Version A Solutions

1. **D** The curve has a vertical tangent when $\frac{dx}{dt} = 2t - 6 = 0$, or $t = 3$. The corresponding point is $(-4, 24)$.

2. **E** Switch x and y and solve for y: $x = \frac{2y+1}{3y-5}$ $3xy - 5x = 2y + 1$, $(3x - 2)y = 5x + 1$, $y = \frac{5x+1}{3x-2}$.

3. **C** $f'(x) = xe^x + e^x = (x+1)e^x$, $f''(x) = (x+1)e^x + e^x = (x+2)e^x$. Proceeding inductively, the 50th derivative is $(x + 50)e^x$.

4. **A** $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{2t-3}{\frac{1}{2}(2t+1)^{-1/2}(2)}$. Setting $\sqrt{2t+1} = 3$ and $t^2 - 3t = 4$ yields $t = 4$. Substituting into the derivative yields $m = \frac{5}{\frac{1}{3}} = 15$.

5. **E** Let y be the distance the car travels and d be the distance between the car and the man. Then $9 + y^2 = d^2$. Differentiate with respect to t: $2y\frac{dy}{dt} = 2d\frac{dd}{dt}$. When $d = 5$, $y = 4$ and $\frac{dy}{dt} = 120$, so $\frac{dd}{dt} = \frac{2(4)(120)}{2(5)} = 96$ ft/sec.

6. **D** $\lim_{x\to 0} \frac{3\sin(2x) + x}{5x} = \lim_{x\to 0} \left(\frac{3\sin(2x)}{5x} + \frac{1}{5}\right)$ $= \lim_{x\to 0} \left(\frac{3 \cdot 2\sin(2x)}{5(2x)} + \frac{1}{5}\right) = \frac{6}{5} + \frac{1}{5} = \frac{7}{5}$.

7. **A** Apply the Chain Rule with $y = \sin u$ and $u = \tan x$: $f'(x) = \cos(\tan(x))\sec^2 x$.

8. **D** $f(2) = L(2) = \frac{3}{2} - 5 = -\frac{7}{2}$ and $f'(2) = L'(2) = -\frac{5}{2}$.

9. **B** As $x \to -\infty$, e^{-x} is the dominating term, so factor this out of the numerator and denominator: $\lim_{x\to-\infty} \frac{e^{-x}(e^{2x}+2)}{e^{-x}(2e^{2x}-1)} = \lim_{x\to-\infty} \frac{e^{2x}+2}{2e^{2x}-1}$ As $x \to -\infty$, $e^{2x} \to 0$, so the limit is $\frac{2}{-1} = -2$.

10. **A** Differentiate implicitly: $3x^2 + 3x\frac{dy}{dx} + 3y + 3y^2\frac{dy}{dx} = 0$. Substitute $x = 2$ and $y = 1$: $12 + 6\frac{dy}{dx} + 3 + 3\frac{dy}{dx} = 0$, and solve for $\frac{dy}{dx} = -\frac{15}{9} = -\frac{5}{3}$.

11. **E** $f'(x) = \frac{1}{2\sqrt{x}}$. $L(x) = f(1) + f'(1)(x-1) = 1 + \frac{1}{2}(x-1)$. Then $L(1.1) = 1 + \frac{1}{2}(1.1-1) = 1 + \frac{1}{20} = \frac{21}{20}$.

12. **C** $g'(2) = \frac{1}{f'(g(2))}$. Since $f(4) = 2$, $g(2) = 4$, so $g'(2) = \frac{1}{f'(4)} = -\frac{1}{2}$

13. .

    (a) $f'(x) = \frac{\tan(\pi x)(6) - (1+6x)(\pi\sec^2(\pi x))}{\tan^2(\pi x)}$

    (b) $g'(x) = -2xe^{-x^2}$ $g''(x) = -2x(-2xe^{-x^2}) + e^{-x^2}(-2) = (4x^2 - 2)e^{-x^2}$

    (c) $y' = e^{ax}(-b\sin(bx)) + \cos(bx)(ae^{ax})$

14. For the first curve $y' = 6x^2$. Implicit differentiation on the second curve yields $2x + 6y\frac{dy}{dx} = 0$, $\frac{dy}{dx} = -\frac{x}{3y}$. At the point $(1, 2)$ the slopes of the tangent lines are 6 for the first curve and $-\frac{1}{6}$ for the second. Since these are negative reciprocals, the curves are orthogonal.

15. $\mathbf{r}'(t) = \langle 6\cos(2t), -8\sin(2t)\rangle$, so the velocity when $t = \frac{\pi}{6}$ is $\mathbf{v} = \left\langle 6\cos\left(\frac{\pi}{3}\right), -8\sin\left(\frac{\pi}{3}\right)\right\rangle = \langle 3, -4\sqrt{3}\rangle$. Speed is $|\mathbf{v}| = \sqrt{3^2 + (4\sqrt{3})^2} = \sqrt{57}$. $\mathbf{r}''(t) = \langle -12\sin(2t), -16\cos(2t)\rangle$, so the acceleration at $t = \frac{\pi}{6}$ is $\mathbf{a} = \langle -6\sqrt{3}, -8\rangle$.

16. We are looking for $\frac{dh}{dt}$ when $h = 200$ and $\frac{dV}{dt} = -9000$. To eliminate r, use similar triangles below to yield $\frac{r}{h} = \frac{200}{600}$, or

$r = \frac{1}{3}h$. Substituting this into our equation yields $V = \frac{1}{3}\pi\left(\frac{1}{3}h\right)^2 h = \frac{1}{27}\pi h^3$. Differentiate and substitute: $\frac{dV}{dt} = \frac{1}{9}\pi h^2 \frac{dh}{dt}$, $-9000 = \frac{1}{9}\pi(200)^2 \frac{dh}{dt}$, so $\frac{dh}{dt} = -\frac{81}{40\pi}$ cm/min.

17. $f'(x) = \frac{1}{2}(4x + (3x-5)^{10})^{-1/2}(4 + 10(3x-5)^9(3))$. Substituting $x = 2$ yields $m = \frac{1}{2}(8+1)^{-1/2}(4 + 10(3)) = \frac{34}{6}$ and $y = \sqrt{8+1} = 3$. Therefore, the equation of the tangent line is $y - 3 = \frac{34}{6}(x-2)$.