Spring 2010 Math 151

Exam II Version B Solutions

1. **B** The curve has a vertical tangent when $\frac{dx}{dt} = 2t + 4 = 0$, or $t = -2$. The corresponding point is $(-1, 21)$.

2. **D** Switch x and y and solve for y: $x = \frac{y+2}{5y-3}$ $5xy - 3x = y + 2$, $(5x-1)y = 3x + 2$, $y = \frac{3x+2}{5x-1}$.

3. **C** $f'(x) = xe^x + e^x = (x+1)e^x$, $f''(x) = (x+1)e^x + e^x = (x+2)e^x$. Proceeding inductively, the 60th derivative is $(x+60)e^x$.

4. **E** $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{\frac{1}{2}(2t+1)^{-1/2}(2)}{2t-3}$. Setting $\sqrt{2t+1} = 3$ and $t^2 - 3t = 4$ yields $t = 4$. Substituting into the derivative yields $m = \frac{\frac{1}{3}}{5} = \frac{1}{15}$.

5. **A** Let y be the distance the car travels and d be the distance between the car and the man. Then $16 + y^2 = d^2$. Differentiate with respect to t: $2y\frac{dy}{dt} = 2d\frac{dd}{dt}$. When $d = 5$, $y = 3$ and $\frac{dy}{dt} = 120$, so $\frac{dd}{dt} = \frac{2(3)(120)}{2(5)} = 72$ ft/sec.

6. **C** $\lim_{x\to 0} \frac{4\sin(2x) + x}{5x} = \lim_{x\to 0}\left(\frac{4\sin(2x)}{5x} + \frac{1}{5}\right)$ $= \lim_{x\to 0}\left(\frac{4\cdot 2\sin(2x)}{5(2x)} + \frac{1}{5}\right) = \frac{8}{5} + \frac{1}{5} = \frac{9}{5}$.

7. **D** Apply the Chain Rule with $y = \tan u$ and $u = \sin x$: $f'(x) = \sec^2(\sin(x))\cos x$.

8. **A** $f(2) = L(2) = \frac{3}{2} + 5 = \frac{13}{2}$ and $f'(2) = L'(2) = \frac{5}{2}$.

9. **E** As $x \to -\infty$, e^{-x} is the dominating term, so factor this out of the numerator and denominator: $\lim_{x\to-\infty} \frac{e^{-x}(2e^{2x}-1)}{e^{-x}(e^{2x}+2)} = \lim_{x\to-\infty}\frac{2e^{2x}-1}{e^{2x}+2}$ As $x \to -\infty$, $e^{2x} \to 0$, so the limit is $\frac{-1}{2} = -\frac{1}{2}$.

10. **C** Differentiate implicitly: $3x^2 + 3x\frac{dy}{dx} + 3y + 3y^2\frac{dy}{dx} = 0$. Substitute $x = 1$ and $y = 2$: $3 + 3\frac{dy}{dx} + 6 + 12\frac{dy}{dx} = 0$, and solve for $\frac{dy}{dx} = -\frac{9}{15} = -\frac{3}{5}$.

11. **D** $f'(x) = \frac{1}{2\sqrt{x}}$. $L(x) = f(1) + f'(1)(x-1) = 1 + \frac{1}{2}(x-1)$. Then $L(1.3) = 1 + \frac{1}{2}(1.3-1) = 1 + \frac{3}{20} = \frac{23}{20}$.

12. **E** $g'(4) = \frac{1}{f'(g(4))}$. Since $f(2) = 4$, $g(4) = 2$, so $g'(4) = \frac{1}{f'(2)} = -\frac{1}{3}$

13. .

   (a) $f'(x) = \frac{\tan(\pi x)(5) - (1+5x)(\pi\sec^2(\pi x))}{\tan^2(\pi x)}$

   (b) $g'(x) = -2xe^{-x^2}$ $g''(x) = -2x(-2xe^{-x^2}) + e^{-x^2}(-2) = (4x^2-2)e^{-x^2}$

   (c) $y' = e^{-ax}(b\cos(bx)) + \sin(bx)(-ae^{-ax})$

14. For the first curve $y' = 6x^2$. Implicit differentiation on the second curve yields $2x + 6y\frac{dy}{dx} = 0$, $\frac{dy}{dx} = -\frac{x}{3y}$. At the point $(1, 2)$ the slopes of the tangent lines are 6 for the first curve and $-\frac{1}{6}$ for the second. Since these are negative reciprocals, the curves are orthogonal.

15. $\mathbf{r}'(t) = \langle 8\cos(2t), -6\sin(2t)\rangle$, so the velocity when $t = \frac{\pi}{6}$ is $\mathbf{v} = \left\langle 8\cos\left(\frac{\pi}{3}\right), -6\sin\left(\frac{\pi}{3}\right)\right\rangle = \langle 4, -3\sqrt{3}\rangle$. Speed is $|\mathbf{v}| = \sqrt{4^2 + (-3\sqrt{3})^2} = \sqrt{43}$. $\mathbf{r}''(t) = \langle -16\sin(2t), -12\cos(2t)\rangle$, so the acceleration at $t = \frac{\pi}{6}$ is $\mathbf{a} = \langle -8\sqrt{3}, -6\rangle$.

16. We are looking for $\frac{dh}{dt}$ when $h = 200$ and $\frac{dV}{dt} = -6000$. To eliminate r, use similar triangles below to yield $\frac{r}{h} = \frac{200}{600}$, or

$r = \frac{1}{3}h$. Substituting this into our equation yields $V = \frac{1}{3}\pi\left(\frac{1}{3}h\right)^2 h = \frac{1}{27}\pi h^3$. Differentiate and substitute: $\frac{dV}{dt} = \frac{1}{9}\pi h^2 \frac{dh}{dt}$, $-6000 = \frac{1}{9}\pi(200)^2\frac{dh}{dt}$, so $\frac{dh}{dt} = -\frac{54}{40\pi}$ cm/min.

17. $f'(x) = \frac{1}{2}(4x + (2x-3)^{10})^{-1/2}(4 + 10(2x-3)^9(2))$. Substituting $x = 2$ yields $m = \frac{1}{2}(8+1)^{-1/2}(4+10(2)) = \frac{24}{6} = 4$ and $y = \sqrt{8+1} = 3$. Therefore, the equation of the tangent line is $y - 3 = 4(x-2)$.