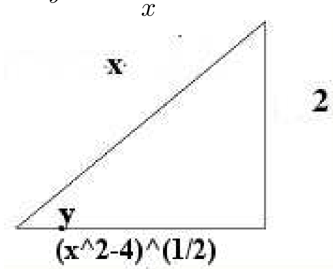
Spring 2010 Math 151

Exam III Version A Solutions

1. **B** $y = \log_9 27$ is equivalent to $9^y = 27$, or $3^{2y} = 3^3$, so $2y = 3$ and $y = \dfrac{3}{2}$.

2. **C** Using properties of logarithms, we have $\log_{10}((5-x)(2-x)) = 1$, which is equivalent to $10 - 7x + x^2 = 10^1$, or $x^2 - 7x = 0$. Then $x(x-7) = 0$, or $x = 0$ and $x = 7$. A quick check shows that $x = 7$ is not in the domain of the original equation, so the only solution is $x = 0$.

3. **D** $y = \sin^{-1}\left(\dfrac{2}{x}\right)$ means $\sin y = \dfrac{2}{x}$. From the reference triangle below, we see that $\cos y = \dfrac{\sqrt{x^2-4}}{x}$.



4. **B** $f'(x) = \dfrac{3x^2}{1+(x^3)^2} = \dfrac{3x^2}{1+x^6}$, so $f'(1) = \dfrac{3}{2}$.

5. **E** The limit is of the indeterminate form $\dfrac{0}{0}$, so apply L'Hospital's Rule: $\displaystyle\lim_{x\to 1} \frac{x^4-1}{2x^3-2} = \lim_{x\to 1}\frac{4x^3}{6x^2} = \frac{2}{3}$.

6. **E** The limit is of the indeterminate form $\infty \cdot 0$, so rewrite as a fraction: $\displaystyle\lim_{x\to(\pi/2)^-} \sec(3x)\cos(x) = \lim_{x\to(\pi/2)^-}\frac{\cos(x)}{\cos(3x)}$ Now apply L'Hospital's Rule: $\displaystyle\lim_{x\to(\pi/2)^-}\frac{-\sin(x)}{-3\sin(3x)} = \frac{-1}{3}$.

7. **B** f is decreasing when f' is negative, which is on the interval $(-\infty,-5)\cup(4,\infty)$.

8. **A** f is concave down when f'' is negative, or when f' is decreasing, which is on the interval $(0,\infty)$.

9. **D** $f'(x) = 3(x-3)^2 = 0$ when $x = 3$. Testing the endpoints and the critical value in the original function yield $f(1) = -8$, $f(3) = 0$, $f(4) = 1$, so the absolute minimum is -8 when $x = 1$.

10. **C** Apply the power rule to the first term and the logarithm rule to the second (NOTE the power rule for x^n only applies when $n \neq -1$). $4\cdot\dfrac{3}{5}x^{5/3} + 3\ln|x| = \dfrac{12}{5}x^{5/3} + 3\ln|x|$.

11. **C** Expand the sum: $\displaystyle\sum_{i=1}^{4} i^2 = 1^2+2^2+3^2+4^2 = 1+4+9+16 = 30.$

12. **E** Since there are only 2 subintervals, the area $= \displaystyle\sum_{i=1}^{2} f(x_i^*)\Delta x_i = f(0)(2) + f(4)(4) = (20)(2) + (4)(4) = 56$.

13. $y = x^{\sin x}\cos x$, so $\ln y = \ln(x^{\sin x}\cos x) = \sin x \ln x + \ln(\cos x)$. Differentiate implicitly: $\dfrac{y'}{y} = \cos x \ln x + \dfrac{\sin x}{x} + \dfrac{-\sin x}{\cos x}$, so $y' = (x^{\sin x}\cos x)\left(\cos x\ln x + \dfrac{\sin x}{x} - \tan x\right)$.

14. Let $y =$the amount of Polonium-218 (in mg) after t days. Then $y = y_0e^{kt} = 1000e^{kt}$. Substitute the amount after 5 days and solve for k: $300 = 1000e^{k(5)}$, $\dfrac{3}{10} = e^{5k}$, $5k = \ln\left(\dfrac{3}{10}\right)$ or $k = \dfrac{1}{5}\ln\left(\dfrac{3}{10}\right)$. After 10 days $1000e^{1/5(\ln(3/10))*10} = 1000e^{2\ln(3/10)} = 1000\left(\dfrac{3}{10}\right)^2 = 90$ mg remain.

15. .

    (a) Find the critical values from the given f': $(x^2-3)e^x = 0$ when $x = \pm\sqrt{3}$. Testing each subinterval, we find f' is positive on $(-\infty,-\sqrt{3})\cup(\sqrt{3},\infty)$ and f' is negative on $(-\sqrt{3},\sqrt{3})$. Therefore, f is increasing on $(-\infty,-\sqrt{3})\cup(\sqrt{3},\infty)$ and decreasing on $(-\sqrt{3},\sqrt{3})$.

    (b) Based on the intervals of increase and decrease, f has a relative maximum at $x = -\sqrt{3}$ and a relative minimum at $x = \sqrt{3}$.

(c) $f''(x) = 2xe^x + (x^2 - 3)e^x = (x^2 + 2x - 3)e^x = (x+3)(x-1)e^x$, which is 0 when $x = -3$ or $x = 1$. Testing each subinterval, we find f'' is positive on $(-\infty, -3) \cup (1, \infty)$ and f'' is negative on $(-3, 1)$. Therefore, f is concave up on $(-\infty, -3) \cup (1, \infty)$ and concave down on $(-3, 1)$.

16. Let ℓ be the length of the field (horizontal) and w be the width. Our goal is to maximize $A = \ell w$ with the restriction that $2\ell + 3w = 3000$. Solve the second equation for ℓ and substitute into the first: $\ell = -\frac{3}{2}w + 1500$, $A =$. $A' = -3w + 1500 = 0$ when $w = 500$. $A'' = -3$, which shows A has a maximum at $w = 500$, $\ell = -\frac{3}{2}(500) + 1500 = 750$. The dimensions which maximize area are 750×500ft.

17. Find the antiderivative of each component to obtain $\mathbf{v}(t) = \langle e^t + \sin t,\, e^t - \cos t\rangle + \mathbf{C}$. If $t = 0$, then $\mathbf{v}(0) = \langle 1, 1\rangle = \langle e^0 + \sin 0, e^0 - \cos 0\rangle + \mathbf{C} = \langle 1, 0\rangle + \mathbf{C}$, so $\mathbf{C} = \langle 0, 1\rangle$. Thus $\mathbf{v}(t) = \langle e^t + \sin t, e^t - \cos t + 1\rangle$. Find the antiderivative again to obtain $\mathbf{r}(t) = \langle e^t - \cos t,\, e^t - \sin t + t\rangle + \mathbf{C}$. If $t = 0$, then $\mathbf{r}(0) = \langle 0, 0\rangle = \langle e^0 - \cos 0,\, e^0 - \sin 0 + 0\rangle + \mathbf{C} = \langle 0, 1\rangle + \mathbf{C}$, so $\mathbf{C} = \langle 0, -1\rangle$. Therefore, $\mathbf{r}(t) = \langle e^t - \cos t, e^t - \sin t + t - 1\rangle$.