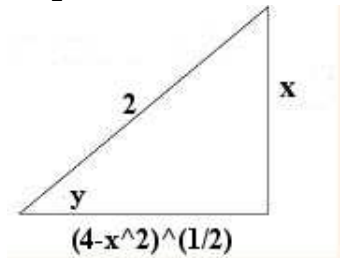


# Spring 2010 Math 151

## Exam III Version B Solutions

- C**  $y = \log_{27} 9$  is equivalent to  $27^y = 9$ , or  $3^{3y} = 3^2$ , so  $3y = 2$  and  $y = \frac{2}{3}$ . (Alternatively, use the change-of-base formula).
- D** Using properties of logarithms, we have  $\log_{10}((5-x)(2-x)) = 1$ , which is equivalent to  $10 - 7x + x^2 = 10^1$ , or  $x^2 - 7x = 0$ . Then  $x(x-7) = 0$ , or  $x = 0$  and  $x = 7$ . A quick check shows that  $x = 7$  is not in the domain of the original equation, so the only solution is  $x = 0$ .
- C**  $y = \sin^{-1}\left(\frac{x}{2}\right)$  means  $\sin y = \frac{x}{2}$ . From the reference triangle below, we see that  $\cos y = \frac{\sqrt{4-x^2}}{2}$ .



- B**  $f'(x) = \frac{3x^2}{1+(x^3)^2} = \frac{3x^2}{1+x^6}$ , so  $f'(1) = \frac{3}{2}$ .
- A** The limit is of the indeterminate form  $\frac{0}{0}$ , so apply L'Hospital's Rule:  $\lim_{x \rightarrow 1} \frac{2x^3 - 2}{x^4 - 1} = \lim_{x \rightarrow 1} \frac{6x^2}{4x^3} = \frac{3}{2}$ .
- D** The limit is of the indeterminate form  $\infty \cdot 0$ , so rewrite as a fraction:  $\lim_{x \rightarrow (\pi/2)^-} \sec(x) \cos(3x) = \lim_{x \rightarrow (\pi/2)^-} \frac{\cos(3x)}{\cos(x)}$ . Now apply L'Hospital's Rule:  $\lim_{x \rightarrow (\pi/2)^-} \frac{-3 \sin(3x)}{-\sin(x)} = \frac{3}{-1}$ .
- C**  $f$  is increasing when  $f'$  is positive, which is on the interval  $(-5, 4)$ .
- E**  $f$  is concave up when  $f''$  is positive, or when  $f'$  is increasing, which is on the interval  $(-\infty, 0)$ .

- A**  $f'(x) = 3(x-3)^2 = 0$  when  $x = 3$ . Testing the endpoints and the critical value in the original function yield  $f(1) = -8$ ,  $f(3) = 0$ ,  $f(4) = 1$ , so the absolute maximum is 1 when  $x = 4$ .

- D** Apply the power rule to the first term and the logarithm rule to the second (NOTE the power rule for  $x^n$  only applies when  $n \neq -1$ ).

$$4 \cdot \frac{3}{5} x^{5/3} + 3 \ln|x| = \frac{12}{5} x^{5/3} + 3 \ln|x|.$$

- E** Expand the sum:

$$\sum_{i=1}^4 i^2 = 1^2 + 2^2 + 3^2 + 4^2 = 1 + 4 + 9 + 16 = 30.$$

- A** Since there are only 2 subintervals, the

$$\text{area} = \sum_{i=1}^2 f(x_i^*) \Delta x_i = f(-2)(2) + f(0)(4) = (16)(2) + (20)(4) = 112.$$

- $y = x^{\cos x} \sin x$ , so  $\ln y = \ln(x^{\cos x} \sin x) = \cos x \ln x + \ln(\sin x)$ . Differentiate implicitly:

$$\frac{y'}{y} = -\sin x \ln x + \frac{\cos x}{x} + \frac{\cos x}{\sin x}, \text{ so } y' = (x^{\cos x} \sin x) \left( -\sin x \ln x + \frac{\cos x}{x} + \cot x \right).$$

- Let  $y$  = the amount of Polonium-218 (in mg) after  $t$  days. Then  $y = y_0 e^{kt} = 1000e^{kt}$ . Substitute the amount after 5 days and solve

$$\text{for } k: 300 = 1000e^{k(5)}, \frac{3}{10} = e^{5k}, 5k = \ln\left(\frac{3}{10}\right) \text{ or } k = \frac{1}{5} \ln\left(\frac{3}{10}\right). \text{ After 15 days } 1000e^{1/5(\ln(3/10)) \cdot 15} = 1000e^{3 \ln(3/10)} = 1000 \left(\frac{3}{10}\right)^3 = 27 \text{ mg remain.}$$

- .

- Find the critical values from the given  $f'$ :  $(x^2 - 8)e^x = 0$  when  $x = \pm\sqrt{8}$ . Testing each subinterval, we find  $f'$  is positive on  $(-\infty, -\sqrt{8}) \cup (\sqrt{8}, \infty)$  and  $f'$  is negative on  $(-\sqrt{8}, \sqrt{8})$ . Therefore,  $f$  is increasing on  $(-\infty, -\sqrt{8}) \cup (\sqrt{8}, \infty)$  and decreasing on  $(-\sqrt{8}, \sqrt{8})$ .

- Based on the intervals of increase and decrease,  $f$  has a relative maximum at  $x = -\sqrt{8}$  and a relative minimum at  $x = \sqrt{8}$ .

(c)  $f''(x) = 2xe^x + (x^2 - 8)e^x = (x^2 + 2x - 8)e^x = (x + 4)(x - 2)e^x$ , which is 0 when  $x = -4$  or  $x = 2$ . Testing each subinterval, we find  $f''$  is positive on  $(-\infty, -4) \cup (2, \infty)$  and  $f''$  is negative on  $(-4, 2)$ . Therefore,  $f$  is concave up on  $(-\infty, -4) \cup (2, \infty)$  and concave down on  $(-4, 2)$ .

16. Let  $\ell$  be the length of the field (horizontal) and  $w$  be the width. Our goal is to maximize  $A = \ell w$  with the restriction that  $2\ell + 3w = 3000$ . Solve the second equation for  $\ell$  and substitute into the first:  $\ell = -\frac{3}{2}w + 1500$ ,  $A = -\frac{3}{2}w^2 + 1500w$ .  $A' = -3w + 1500 = 0$  when  $w = 500$ .  $A'' = -3$ , which shows  $A$  has a maximum at  $w = 500$ ,  $\ell = -\frac{3}{2}(500) + 1500 = 750$ . The dimensions which maximize area are  $750 \times 500$ ft.
17. Find the antiderivative of each component to obtain  $\mathbf{v}(t) = \langle e^t - \cos t, e^t + \sin t \rangle + \mathbf{C}$ . If  $t = 0$ , then  $\mathbf{v}(0) = \langle 1, 1 \rangle = \langle e^0 - \cos 0, e^0 + \sin 0 \rangle + \mathbf{C} = \langle 0, 1 \rangle + \mathbf{C}$ , so  $\mathbf{C} = \langle 1, 0 \rangle$ . Thus  $\mathbf{v}(t) = \langle e^t - \cos t + 1, e^t + \sin t \rangle$ . Find the antiderivative again to obtain  $\mathbf{r}(t) = \langle e^t - \sin t + t, e^t - \cos t \rangle + \mathbf{C}$ . If  $t = 0$ , then  $\mathbf{r}(0) = \langle 0, 0 \rangle = \langle e^0 - \sin 0 + 0, e^0 - \cos 0 \rangle + \mathbf{C} = \langle 1, 0 \rangle + \mathbf{C}$ , so  $\mathbf{C} = \langle -1, 0 \rangle$ . Therefore,  $\mathbf{r}(t) = \langle e^t - \sin t + t - 1, e^t - \cos t \rangle$ .