

**MATH 151, FALL 2010  
COMMON EXAM III - VERSION A**

LAST NAME, First name (print): \_\_\_\_\_

INSTRUCTOR: \_\_\_\_\_

SECTION NUMBER: \_\_\_\_\_

UIN: \_\_\_\_\_

SEAT NUMBER: \_\_\_\_\_

**DIRECTIONS:**

1. The use of a calculator, laptop or computer is prohibited.
2. In Part 1 (Problems 1-10), mark the correct choice on your ScanTron using a No. 2 pencil. The scantrons will not be returned, therefore *for your own records, also record your choices on your exam!*
3. In Part 2 (Problems 11-16), present your solutions in the space provided. *Show all your work* neatly and concisely and *clearly indicate your final answer*. You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.
4. Be sure to *write your name, section number and version letter of the exam on the ScanTron form.*

THE AGGIE CODE OF HONOR

**“An Aggie does not lie, cheat or steal, or tolerate those who do.”**

Signature: \_\_\_\_\_

**DO NOT WRITE BELOW!**

Question	Points Awarded	Points
1-10		40
11		12
12		8
13		10
14		10
15		10
16		10
		100

PART I: Multiple Choice

1. (4 pts)  $\lim_{t \rightarrow \frac{\pi}{2}^-} \ln(\cos t) =$

- (a) 0
- (b)  $-\infty$
- (c) 1
- (d)  $e$
- (e) -1

2. (4 pts) If it is given that  $f'(x) = 4 - \frac{5}{x^2 + 1}$  and  $f(1) = 0$ , find  $f(x)$ .

- (a)  $f(x) = 4x - \frac{5}{2} \ln(x^2 + 1) + \frac{5}{2} \ln 2 - 4$
- (b)  $f(x) = 4x - 5 \arctan x - 4$
- (c)  $f(x) = 4x - 5 \ln(x^2 + 1) + 5 \ln 2 - 4$
- (d)  $f(x) = 4x - 5 \arctan x + \frac{5\pi}{4} - 4$
- (e)  $f(x) = 4x - \frac{5}{2} \arctan x + \frac{5\pi}{8} - 4$

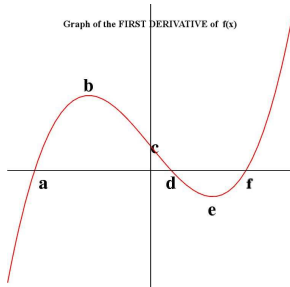
3. (4 pts)  $\lim_{x \rightarrow 0} (1 - 4x)^{1/x} =$

- (a) 4
- (b) -4
- (c)  $e^4$
- (d)  $e^{-4}$
- (e) 1

4. (4 pts) Find all critical numbers for  $f(x) = |x^2 + 4x - 12|$  on the interval  $(-\infty, \infty)$ .

- (a)  $x = -2$
- (b)  $x = -6$  and  $x = 2$
- (c)  $x = -2, x = 6$  and  $x = 2$
- (d)  $x = -2$  and  $x = 0$
- (e)  $x = -2, x = -6$  and  $x = 2$

5. (4 pts) The graph of the *DERIVATIVE* of  $f(x)$  is shown below. Where does  $f(x)$  change concavity?



- (a)  $a, d$  and  $f$
- (b)  $b$  and  $e$
- (c)  $c$
- (d)  $a$  and  $f$
- (e)  $a$

6. (4 pts) What is the domain of  $f(t) = \arccos(3 + t)$ ?

- (a)  $-3 \leq t \leq \pi - 3$
- (b)  $-4 < t < -2$
- (c)  $-3 < t < \pi - 3$
- (d)  $-4 \leq t \leq -2$
- (e)  $t \geq 0$

7. (4 pts) Where is  $f(x) = e^{x^3 - 3x}$  increasing?

- (a) On  $(1, \infty)$  only
- (b) On  $(-1, 1)$  only
- (c) On  $(-\infty, -1)$  and  $(1, \infty)$  only
- (d) On  $(0, \infty)$  only
- (e) On  $(-\infty, \infty)$

8. (4 pts) Find the derivative of  $f(x) = \log_{10}(x^2 + 2^x)$ .

(a)  $\frac{2x}{(x^2 + 2^x) \ln 10}$

(b)  $\frac{2x + 2^x}{(x^2 + 2^x) \ln 10}$

(c)  $\frac{2x + 2^x \ln 2}{(x^2 + 2^x)}$

(d)  $\frac{2x + 2^x \ln 2}{(x^2 + 2^x) \ln 10}$

(e)  $\frac{2}{x^2 \ln 10} + \log_{10}(2)$

9. (4 pts) Where is  $f(x) = \ln(x^2 + 1)$  concave up?

(a) On  $(1, \infty)$  only

(b) On  $(-1, 1)$  only

(c) On  $(-\infty, -1)$  and  $(1, \infty)$  only

(d) On  $(-\infty, -1)$  only

(e) On  $(-\infty, \infty)$

10. (4 pts) Find the absolute extrema for  $f(x) = 12 - x - \frac{9}{x}$  on the interval  $[1, 4]$ .

(a) The absolute maximum value is 6 and the absolute minimum value is 2.

(b) The absolute maximum value is  $\frac{23}{4}$  and the absolute minimum value is 2.

(c) The absolute maximum value is 6 and the absolute minimum value is 0.

(d) The absolute maximum value is  $\frac{23}{4}$  and the absolute minimum value is 0.

(e) The absolute maximum value is 18 and the absolute minimum value is 2.

## PART II WORK OUT

**Directions:** Present your solutions in the space provided. *Show all your work* neatly and concisely and *Box your final answer*. You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.

11. (12 pts total)

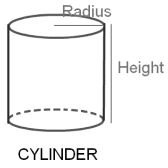
(i) (9 pts) Find the inverse of the function  $f(x) = \frac{3e^{5x-1}}{1 + e^{5x-1}}$ .

(ii) (3 pts) Find the domain of the inverse.

12. (8 pts) An object has an acceleration of  $\mathbf{a}(t) = t^3\mathbf{i} + 2\sin(t)\mathbf{j}$ , an initial velocity of  $\mathbf{v}(0) = \mathbf{j}$ , and an initial position of  $\mathbf{s}(0) = 3\mathbf{i}$ . Find a vector function that describes the position of the object at time  $t$ .

13. (10 pts) If  $y$  is a differentiable function of  $x$  on the interval  $(0, \infty)$ , find a formula for  $\frac{dy}{dx}$  if  $x^y = y^x$ .

14. (10 pts) A soft drink company wants to make a new drink and sell it in a cylindrical shaped can. The can must hold  $12\pi$  cubic inches of soda. The construction of this can costs the company \$0.50 per square inch to make the top of the can, \$1.50 per square inch to make the bottom of the can, and \$1.00 per square inch to make the sides of the can. Use calculus to determine the radius and height of the can which would minimize the company's cost to make the can.



15. (10 pts) A tank contains 1000 L of sugar water with 8 kg of dissolved sugar. Pure water enters the tank at a rate of 4 liters per minute. The solution is kept mixed and exits the tank at the same rate. How much sugar (in kg) is in the tank after 7 minutes?

16. (10 pts) Let  $C$  denote the graph of  $y = \arcsin x$ ,  $-1 \leq x \leq 1$ . Determine the point(s) on  $C$  where the tangent line to  $C$  is parallel to the line  $y = 2x$ .