

Spring 2011 Math 151

Exam I Version B Solutions

1. **C** Set x and y components equal and solve for t . $t = 4$ at $(2, -3)$, $t = 0$ at $(0, 1)$, $t = 1$ at $(1, 0)$, but $-1 = \sqrt{t}$ has no solution, so the point not on the curve is $(-1, 0)$.

2. **A**
$$\frac{\cos x}{1 - \sin x} \cdot \frac{(1 + \sin x)}{(1 + \sin x)} = \frac{\cos x(1 + \sin x)}{1 - \sin^2 x} = \frac{1 + \sin x}{\cos x} = \sec x + \tan x.$$

3. **A** Since $-1 \leq \cos x \leq 1$, $-\frac{1}{x} \leq \frac{\cos x}{x} \leq \frac{1}{x}$. Since $\pm \frac{1}{x} \rightarrow 0$ as $x \rightarrow \infty$, by the Squeeze Theorem, the limit is **0**.

4. **B** As we approach 4 from the left along the curve, the y -value approaches **2**.

5. **B** Let $f(x) = x^3 - x^2 + x$. f is continuous since it is a polynomial, and $f(2) = 6$, $f(3) = 21$, so $f(2) < 10 < f(3)$. Therefore, **by the Intermediate Value Theorem, there is a solution to $f(x) = 10$ on $[2, 3]$.**

6. **C** $\mathbf{a} = (6\mathbf{i} + 3\mathbf{j}) - (3\mathbf{i} - \mathbf{j}) = 3\mathbf{i} + 4\mathbf{j}$. To form a unit vector $\hat{\mathbf{a}}$, multiply by the reciprocal of the magnitude: $\hat{\mathbf{a}} = \frac{1}{\sqrt{3^2 + 4^2}}(3\mathbf{i} + 4\mathbf{j}) = \frac{3}{5}\mathbf{i} + \frac{4}{5}\mathbf{j}$.

7. **D**
$$\lim_{x \rightarrow 5^-} f(x) = 6 - 5 = 1.$$

$$\lim_{x \rightarrow 5^+} f(x) = -8 + 2(5) = 2, \quad \text{and} \quad f(5) = 2.$$
 Therefore, since $\lim_{x \rightarrow 5^+} f(x) = f(5) \neq \lim_{x \rightarrow 5^-} f(x)$, **f is continuous only from the right.**

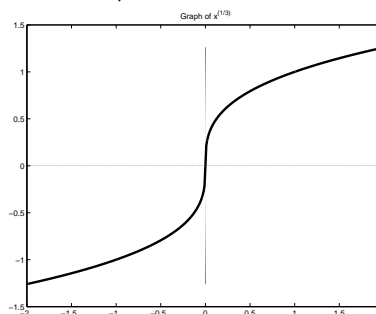
8. **B** Factor x from numerator and denominator:
$$\lim_{x \rightarrow \infty} \frac{x(3 - \frac{8}{x})}{x(4 + \frac{9}{x})} = \frac{3}{4}.$$
 Similarly,
$$\lim_{x \rightarrow -\infty} \frac{x(3 - \frac{8}{x})}{x(4 + \frac{9}{x})} = \frac{3}{4}.$$

9. **E** The angle between the gravity force (weight) and the motion of the block is 60° , so the work done is $W = |\mathbf{F}||\mathbf{D}| \cos 60^\circ = (30)(20) \left(\frac{1}{2}\right) = \mathbf{300 \text{ ft-lbs}}$.

10. **B** $f'(3) = \lim_{h \rightarrow 0} \frac{f(3+h) - f(3)}{h} = \lim_{h \rightarrow 0} h^2 - 4h + 7 = 7.$ The equation of the line whose slope is 7 and which passes through the point $(3, 4)$ is $y - 4 = 7(x - 3)$, or **$y = 7x - 17$.**

11. **E** $\lim_{x \rightarrow 6} \frac{x(x-6)}{x-6} = \lim_{x \rightarrow 6} x = \mathbf{6}.$

12. **E** (a) is both continuous and differentiable everywhere; (c) and (d) are not continuous at $x = 0$. From the graph of (e), we see that the function continuous but not differentiable at $x = 0$ is $\sqrt[3]{x}$.



13. $f'(1) = \lim_{x \rightarrow 1} \frac{f(x) - f(1)}{x - 1} = \lim_{h \rightarrow 0} \frac{\sqrt{2+3x} - \sqrt{5}}{x - 1} \cdot \frac{\sqrt{2+3x} + \sqrt{5}}{\sqrt{2+3x} + \sqrt{5}}$
 $= \lim_{x \rightarrow 1} \frac{2 + 3x - 5}{(x - 1)(\sqrt{2+3x} + \sqrt{5})}$
 $= \lim_{x \rightarrow 1} \frac{3(x - 1)}{(x - 1)(\sqrt{2+3x} + \sqrt{5})} = \lim_{x \rightarrow 1} \frac{3}{\sqrt{2+3x} + \sqrt{5}} = \frac{3}{2\sqrt{5}}.$

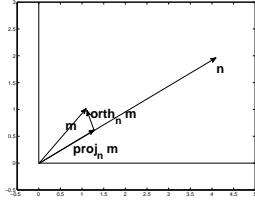
14. .

(a) $\text{proj}_{\mathbf{n}} \mathbf{m} = \frac{\mathbf{m} \cdot \mathbf{n}}{|\mathbf{n}|^2} \mathbf{n} = \frac{4 + 2}{(4^2 + 2^2)} \langle 4, 2 \rangle$
 $= \left\langle \frac{6}{5}, \frac{3}{5} \right\rangle$, so $\text{orth}_{\mathbf{n}} \mathbf{m} = \langle 1, 1 \rangle - \left\langle \frac{6}{5}, \frac{3}{5} \right\rangle = \left\langle -\frac{1}{5}, \frac{2}{5} \right\rangle.$

- (b) The point $(0, 0)$ is on the line, so let $\mathbf{b} = \langle 1, 1 \rangle - \langle 0, 0 \rangle = \langle 1, 1 \rangle$. The vector $\mathbf{v} = \langle 4, 2 \rangle$ is in the direction of the line, so $\mathbf{v}^\perp = \langle -2, 4 \rangle$ is orthogonal to the line. The distance is found

$$\text{by } |\text{comp}_{\mathbf{v}^\perp} \mathbf{b}| = \frac{-2+4}{\sqrt{(-2)^2+4^2}} = \frac{2}{\sqrt{20}}$$

(NOTE that this is the magnitude of the vector in (a) above!)



15. Find a common denominator:

$$\begin{aligned} \lim_{x \rightarrow 2} \frac{\frac{1}{x} \cdot \frac{2}{2} - \frac{1}{2} \cdot \frac{x}{x}}{x-2} &= \lim_{x \rightarrow 2} \frac{1}{x-2} \cdot \frac{2-x}{2x} \\ &= \lim_{x \rightarrow 2} -\frac{1}{2x} = -\frac{1}{4}. \end{aligned}$$

16. $\mathbf{F}_1 = \langle 8 \cos 45^\circ, 8 \sin 45^\circ \rangle = \langle 4\sqrt{2}, 4\sqrt{2} \rangle$.
 $\mathbf{F}_2 = \langle 14 \cos 60^\circ, -14 \sin 60^\circ \rangle = \langle 7, -7\sqrt{3} \rangle$.
 The resultant force is $\mathbf{F}_1 + \mathbf{F}_2 = \langle 4\sqrt{2} + 7, 4\sqrt{2} - 7\sqrt{3} \rangle$. Therefore, the magnitude of the resultant force is $\sqrt{(4\sqrt{2} + 7)^2 + (4\sqrt{2} - 7\sqrt{3})^2}$.

17. We use $\mathbf{r}(t) = \mathbf{r}_0 + t\mathbf{v}$ to find the parametric equations. \mathbf{r}_0 corresponds to the point at $t = 1$: $\mathbf{r}(1) = 5\mathbf{i} + 5\mathbf{j}$. $\mathbf{v} = \mathbf{r}'(1) = \lim_{t \rightarrow 1} \frac{((5t)\mathbf{i} + (8 - 3t^2)\mathbf{j}) - (5\mathbf{i} + 5\mathbf{j})}{t-1} = \lim_{t \rightarrow 1} \frac{(5t-5)\mathbf{i} + (3-3t^2)\mathbf{j}}{t-1} = \lim_{t \rightarrow 1} \left(\frac{5t-5}{t-1} \right) \mathbf{i} + \left(\frac{3-3t^2}{t-1} \right) \mathbf{j} = \left(\lim_{t \rightarrow 1} \frac{5(t-1)}{t-1} \right) \mathbf{i} + \left(\lim_{t \rightarrow 1} \frac{3(1+t)(1-t)}{t-1} \right) \mathbf{j} = 5\mathbf{i} - 6\mathbf{j}$. Therefore, the vector equation of the tangent line is $\mathbf{r}(t) = (5\mathbf{i} + 5\mathbf{j}) + t(5\mathbf{i} - 6\mathbf{j})$, so the parametric equations are $\mathbf{r}(t) = (5 + 5t)\mathbf{i} + (5 - 6t)\mathbf{j}$, or $\mathbf{x} = 5 + 5t$, $\mathbf{y} = 5 - 6t$.