Spring 2011 Math 151

Exam II Version A Solutions

1. **A** Differentiate to find the velocity: $s'(t) = 6t^2 - 42t + 72$, so $s'(1) = \mathbf{36}$.

2. **C** Differentiate to find the velocity: $\mathbf{r}'(t) = \langle 1+e^t, 1+2t\rangle$. At the point $(1,0)$, $t+e^t = 1$ and $t+t^2 = 0$, so $t = 0$. The velocity is $\mathbf{r}'(0) = \langle 2,1\rangle$, so the speed is $|\mathbf{r}'(0)| = \sqrt{\mathbf{5}}$.

3. **C** If you don't just remember the key limit, multiply by the conjugate: $\lim_{x\to 0} \frac{1-\cos x}{2x}\cdot\frac{1+\cos x}{1+\cos x}$ $= \lim_{x\to 0}\frac{1-\cos^2 x}{2x(1+\cos x)} = \lim_{x\to 0}\frac{\sin^2 x}{2x(1+\cos x)}$ $= \lim_{x\to 0}\frac{\sin x}{x}\cdot\frac{\sin x}{2(1+\cos x)} = 1\cdot 0 = \mathbf{0}$.

4. **C** Switch x and y and solve for y: $x = \sqrt{y-5}$, so $x^2 = y-5$ and $y = \mathbf{x^2+5}$. The domain of f^{-1} is the range of f, which is $[\mathbf{0}, \infty)$.

5. **B** $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{3t^2}{2t-4}$. When $t=2$, $\frac{dy}{dt} = 12$ and $\frac{dx}{dt} = 0$, so the graph has a vertical tangent at $t=2$. When $t=2$, $x = 2^2 - 4(2)+1 = -3$, so the equation is $\mathbf{x = -3}$.

6. **B** $Q(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2}(x-a)^2$. $f(x) = \cos(2x)$, $f'(x) = -2\sin(2x)$, $f''(x) = -4\cos(2x)$, so $Q(x) = \cos\left(2\cdot\frac{\pi}{2}\right) - 2\sin\left(2\cdot\frac{\pi}{2}\right)\left(x-\frac{\pi}{2}\right) - 2\cos$ $= \mathbf{-1-2\left(x-\frac{\pi}{2}\right)^2}$.

7. **E** Apply the Quotient Rule: $f'(x) = \frac{(2x+3)(4)-(4x+1)(2)}{(2x+3)^2}$, so $f'(1) = \frac{(5)(4)-(5)(2)}{(5)^2} = \frac{10}{25} = \mathbf{\frac{2}{5}}$

8. **B** $g'(x) = xf'(x) + f(x)(1)$, so $m = g'(2) = 2f'(2)+f(2) = (2)(-5)+3 = -7$. $g(2) = (2)f(2) = (2)(3) = 6$, so the equation of the tangent line is $\mathbf{y-6 = -7(x-2)}$.

9. **D** $f'(x) = 3(x^2+1)^2(2x)$, so $f'(1) = 3(2)^2(2) = \mathbf{24}$.

10. **D** Look at the exponent first: $\lim_{x\to 2^+}\frac{1}{2-x} = -\infty$. Therefore, $\lim_{x\to 2^+} 3^{1/(2-x)} = 0$ (as the exponent approaches $-\infty$, the exponential approaches $\mathbf{0}$.

11. **E** The solution is of the form $y = e^{rx}$, where r is an unknown constant. Differentiate, substitute, and solve: $y' = re^{rx}$, $y'' = r^2e^{rx}$. Then $r^2e^{rx} + re^{rx} - 6e^{rx} = 0$. $e^{rx}(r^2+r-6) = e^{rx}(r+3)(r-2) = 0$. Since $e^{rx}\neq 0$, $r=-3$ and $r=2$. The solutions are $y = e^{-3x}$ and $y = e^{2x}$, which means **none of these is correct**.

12. **A** $g'(2) = \frac{1}{f'(g(2))}$. $f'(x) = 3x^2+1$. To find $g(2)$, let $y = g(2)$. Then $2 = f(y) = y^3+y$. By inspection, $y^3+y = 2$ when $y=1$. So $g'(2) = \frac{1}{f'(1)} = \mathbf{\frac{1}{4}}$

13. Let x be the distance from the bottom of the ladder to the wall and y be the distance from the top of the ladder to the floor. Then $x^2+y^2 = 100$. When $x=6$, $\frac{dx}{dt} = 1$ and $y = \sqrt{100-36} = 8$. Differentiate, substitute, and solve: $2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$, $2(6)(1) + 2(8)\frac{dy}{dt} = 0$, so $\frac{dy}{dt} = -\frac{3}{4}$ ft/s, which means the ladder is sliding down the wall at a speed of $\mathbf{\frac{3}{4}}$ **ft/sec**.

14. .

    (a) $f'(x) = \mathbf{2x\cot(3x) - x^2(3\csc^2(3x))}$.

    (b) $g'(x) = \dfrac{\mathbf{(\sqrt{x}+x^2)(\frac{1}{2}x^{-1/2}e^{\sqrt{x}}) - e^{\sqrt{x}}(\frac{1}{2}x^{-1/2}+2x)}}{\mathbf{(\sqrt{x}+x^2)^2}}$

    (c) $u'(x) = \mathbf{3\tan^2(e^{-x}-ex+e^2)\sec^2(e^{-x}-ex+e^2)(-e^{-x}-e)}$

15. $\mathbf{r}'(t) = (8\cos^2 t(-\sin t))\mathbf{i} + (8\sin^2 t(\cos t))\mathbf{j}$. $\mathbf{r}'\left(\frac{\pi}{6}\right) = \left(-8\cos^2\left(\frac{\pi}{6}\right)\sin\left(\frac{\pi}{6}\right)\right)\mathbf{i} + \left(8\sin^2\left(\frac{\pi}{6}\right)\cos\left(\frac{\pi}{6}\right)\right)\mathbf{j} = \left(-8\cdot\frac{3}{4}\cdot\frac{1}{2}\right)\mathbf{i} + \left(8\cdot\frac{1}{4}\cdot\frac{\sqrt{3}}{2}\right)\mathbf{j} = -3\mathbf{i} + \sqrt{3}\mathbf{j}$. To form a unit tangent vector, multiply by the reciprocal of the magnitude: $\mathbf{T}\left(\frac{\pi}{6}\right) = \frac{1}{\sqrt{9+3}}(-3\mathbf{i} + \sqrt{3}\mathbf{j}) = -\frac{\mathbf{3}}{\sqrt{\mathbf{12}}}\mathbf{i} + \frac{\sqrt{\mathbf{3}}}{\sqrt{\mathbf{12}}}\mathbf{j}$.

16. $f'(x) = -1(1-x)^{-2}(-1) = (1-x)^{-2}$. $f''(x) = -(2\cdot 1)(1-x)^{-3}(-1) = (2\cdot 1)(1-x)^{-3}$. $f'''(x) = -(3\cdot 2\cdot 1)(1-x)^{-4}(-1) = (3\cdot 2\cdot 1)(1-x)^{-4}$ $f^{iv}(x) = -(4\cdot 3\cdot 2\cdot 1)(1-x)^{-5}(-1) = (4\cdot 3\cdot 2\cdot 1)(1-x)^{-5}$. Following this pattern, $f^{(n)}(x) = \mathbf{n!(1-x)^{-(n+1)}}$.

17. .

(a) Differentiate implicitly: $3S^2\frac{dS}{dT} + \frac{1}{2}\frac{dS}{dT} = 2T$, so $\frac{dS}{dT}\left(3S^2 + \frac{1}{2}\right) = 2T$ and $\frac{dS}{dT} = \frac{\mathbf{2T}}{\mathbf{3S^2 + \frac{1}{2}}}$.

(b) Substitute: $2^3 + \frac{1}{2}(2) = 3^2$, $8 + 1 = 9$

(c) $dS = S'(T)dT = \frac{2T}{3S^2 + \frac{1}{2}}dT$. When $T = 3$, $S = 2$, and $dT = \frac{1}{10}$, so $dS = \frac{2(3)}{3(2)^2 + \frac{1}{2}}\cdot\frac{1}{10} = \frac{6}{125}$. Therefore, The approximation to S is $\mathbf{2} + \frac{\mathbf{6}}{\mathbf{125}}$, or $\mathbf{2.048}$.