

Spring 2011 Math 151

Exam II Version B Solutions

- E** Differentiate to find the velocity: $s'(t) = 6t^2 - 42t + 72$, so $s'(1) = \mathbf{36}$.
- D** If you don't just remember the key limit, multiply by the conjugate:

$$\lim_{x \rightarrow 0} \frac{1 - \cos x}{2x} \cdot \frac{1 + \cos x}{1 + \cos x}$$

$$= \lim_{x \rightarrow 0} \frac{1 - \cos^2 x}{2x(1 + \cos x)} = \lim_{x \rightarrow 0} \frac{\sin^2 x}{2x(1 + \cos x)}$$

$$= \lim_{x \rightarrow 0} \frac{\sin x}{x} \cdot \frac{\sin x}{2(1 + \cos x)} = 1 \cdot 0 = \mathbf{0}.$$
- A** $g'(2) = \frac{1}{f'(g(2))}$. $f'(x) = 3x^2 + 1$. To find $g(2)$, let $y = g(2)$. Then $2 = f(y) = y^3 + y$. By inspection, $y^3 + y = 2$ when $y = 1$. So $g'(2) = \frac{1}{f'(1)} = \frac{1}{4}$.
- B** Switch x and y and solve for y : $x = \sqrt{y-5}$, so $x^2 = y - 5$ and $y = x^2 + 5$. The domain of f^{-1} is the range of f , which is $[0, \infty)$.
- D** $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{3t^2}{2t-4}$. When $t = 2$, $\frac{dy}{dx} = 12$ and $\frac{dx}{dt} = 0$, so the graph has a vertical tangent at $t = 2$. When $t = 2$, $x = 2^2 - 4(2) + 1 = -3$, so the equation is $\mathbf{x = -3}$.
- C** Differentiate to find the velocity: $\mathbf{r}'(t) = \langle 1 + e^t, 1 + 2t \rangle$. At the point $(1, 0)$, $t + e^t = 1$ and $t + t^2 = 0$, so $t = 0$. The velocity is $\mathbf{r}'(0) = \langle 2, 1 \rangle$, so the speed is $|\mathbf{r}'(0)| = \sqrt{5}$.
- B** $Q(x) = f(a) + f'(a)(x - a) + \frac{f''(a)}{2}(x - a)^2$. $f(x) = \cos(2x)$, $f'(x) = -2\sin(2x)$, $f''(x) = -4\cos(2x)$, so $Q(x) = \cos\left(2 \cdot \frac{\pi}{2}\right) - 2\sin\left(2 \cdot \frac{\pi}{2}\right)\left(x - \frac{\pi}{2}\right) - 2\cos\left(x - \frac{\pi}{2}\right)^2 = \mathbf{-1 - 2\left(x - \frac{\pi}{2}\right)^2}$.
- C** Apply the Quotient Rule: $f'(x) = \frac{(2x+3)(4) - (4x+1)(2)}{(2x+3)^2}$, so $f'(1) = \frac{(5)(4) - (5)(2)}{(5)^2} = \frac{10}{25} = \frac{2}{5}$.
- B** $g'(x) = xf'(x) + f(x)(1)$, so $m = g'(2) = 2f'(2) + f(2) = (2)(-5) + 3 = -7$. $g(2) = (2)f(2) = (2)(3) = 6$, so the equation of the tangent line is $\mathbf{y - 6 = -7(x - 2)}$.
- E** The solution is of the form $y = e^{rx}$, where r is an unknown constant. Differentiate, substitute, and solve: $y' = re^{rx}$, $y'' = r^2e^{rx}$. Then $r^2e^{rx} + re^{rx} - 6e^{rx} = 0$. $e^{rx}(r^2 + r - 6) = e^{rx}(r + 3)(r - 2) = 0$. Since $e^{rx} \neq 0$, $r = -3$ and $r = 2$. The solutions are $y = e^{-3x}$ and $y = e^{2x}$, which means **none of these is correct**.
- C** Look at the exponent first: $\lim_{x \rightarrow 2^+} \frac{1}{2-x} = -\infty$. Therefore, $\lim_{x \rightarrow 2^+} 3^{1/(2-x)} = 0$ (as the exponent approaches $-\infty$, the exponential approaches $\mathbf{0}$).
- A** $f'(x) = 3(x^2 + 1)^2(2x)$, so $f'(1) = 3(2)^2(2) = \mathbf{24}$.
- Let x be the distance from the bottom of the ladder to the wall and y be the distance from the top of the ladder to the floor. Then $x^2 + y^2 = 100$. When $x = 6$, $\frac{dx}{dt} = 1$ and $y = \sqrt{100 - 36} = 8$. Differentiate, substitute, and solve: $2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$, $2(6)(1) + 2(8)\frac{dy}{dt} = 0$, so $\frac{dy}{dt} = -\frac{3}{4}$ ft/s, which means the ladder is sliding down the wall at a speed of $\frac{3}{4}$ ft/sec.
- .
 - $f'(x) = \mathbf{2x \cot(3x) - x^2(3 \csc^2(3x))}$.
 - $g'(x) = \frac{(\sqrt{x} + x^2)(\frac{1}{2}x^{-1/2}e^{\sqrt{x}}) - e^{\sqrt{x}}(\frac{1}{2}x^{-1/2} + 2x)}{(\sqrt{x} + x^2)^2}$
 - $u'(x) = \frac{3 \tan^2(e^{-x} - ex + e^2) \sec^2(e^{-x} - ex + e^2)(-e^{-x} - e)}{}$

15. $\mathbf{r}'(t) = (8 \cos^2 t(-\sin t))\mathbf{i} + (8 \sin^2 t(\cos t))\mathbf{j}$.
 $\mathbf{r}'\left(\frac{\pi}{6}\right) = \left(-8 \cos^2\left(\frac{\pi}{6}\right) \sin\left(\frac{\pi}{6}\right)\right)\mathbf{i} + \left(8 \sin^2\left(\frac{\pi}{6}\right) \cos\left(\frac{\pi}{6}\right)\right)\mathbf{j} = \left(-8 \cdot \frac{3}{4} \cdot \frac{1}{2}\right)\mathbf{i} + \left(8 \cdot \frac{1}{4} \cdot \frac{\sqrt{3}}{2}\right)\mathbf{j} = -3\mathbf{i} + \sqrt{3}\mathbf{j}$. To form a unit tangent vector, multiply by the reciprocal of the magnitude: $\mathbf{T}\left(\frac{\pi}{6}\right) = \frac{1}{\sqrt{9+3}}(-3\mathbf{i} + \sqrt{3}\mathbf{j}) = -\frac{3}{\sqrt{12}}\mathbf{i} + \frac{\sqrt{3}}{\sqrt{12}}\mathbf{j}$.

16. $f'(x) = -1(1-x)^{-2}(-1) = (1-x)^{-2}$.
 $f''(x) = -(2 \cdot 1)(1-x)^{-3}(-1) = (2 \cdot 1)(1-x)^{-3}$.
 $f'''(x) = -(3 \cdot 2 \cdot 1)(1-x)^{-4}(-1) = (3 \cdot 2 \cdot 1)(1-x)^{-4}$.
 $f^{iv}(x) = -(4 \cdot 3 \cdot 2 \cdot 1)(1-x)^{-5}(-1) = (4 \cdot 3 \cdot 2 \cdot 1)(1-x)^{-5}$.
 Following this pattern, $f^{(n)}(x) = \mathbf{n!}(1-x)^{-(n+1)}$.

17. .

(a) Differentiate implicitly: $3S^2 \frac{dS}{dT} + \frac{1}{2} \frac{dS}{dT} = 2T$, so $\frac{dS}{dT} \left(3S^2 + \frac{1}{2}\right) = 2T$
 and $\frac{dS}{dT} = \frac{2T}{3S^2 + \frac{1}{2}}$.

(b) Substitute: $2^3 + \frac{1}{2}(2) = 3^2$, $8 + 1 = 9$

(c) $dS = S'(T)dT = \frac{2T}{3S^2 + \frac{1}{2}}dT$. When $T = 3$, $S = 2$, and $dT = \frac{1}{10}$, so $dS = \frac{2(3)}{3(2)^2 + \frac{1}{2}} \cdot \frac{1}{10} = \frac{6}{125}$. Therefore, The approximation to S is $2 + \frac{6}{125}$, or **2.048**.