Spring 2011 Math 151

Exam III Version A Solutions

1. **D** Since the numerator and denominator both approach 0, use L'Hospital's Rule: $\lim_{x\to 0}\frac{e^x - \cos x - 2x}{x^2-2x} = \lim_{x\to 0}\frac{e^x+\sin x - 2}{2x-2} = \mathbf{\frac{1}{2}}$.

2. **E** Use properties of logarithms: $\ln(x^2+x) = \ln(x+4)$, so $x^2+x = x+4$, $x^2-4=0$, which yields $x=2$ or $x=-2$. Since the domain of the left-hand side of the original equation is $x>0$, the only solution is $\mathbf{x=2}$.

3. **A** f' positive means f is increasing, and f' decreasing means f'' is negative, so f is concave down. The only graph increasing and concave down is graph **A**.

4. **B** The original function f is decreasing when f' is negative, which occurs when $x \in (\mathbf{a}, \mathbf{c}) \cup (\mathbf{e}, \infty)$.

5. **B** f has a critical value when $f'=0$, namely when $x = a, c, e$. Using the signs of the derivative f', we find that f is increasing for $x \in (-\infty, a)\cup(c,e)$ and decreasing for $x \in (a,c)\cup(e,\infty)$. Therefore, f has a local minimum only when $\mathbf{x=c}$.

6. **E** Let $y = \log_4\left(\frac{1}{8}\right)$. Then $4^y = \frac{1}{8}$, or $2^{2y} = 2^{-3}$, which means $2y=-3$ and $y = -\mathbf{\frac{3}{2}}$. (Alternatively, $\log_4\left(\frac{1}{8}\right) = \frac{\ln 1/8}{\ln 4} = \frac{-3\ln 2}{2\ln 2}$ and cancel)

7. **A** Exponential growth means $y = Ce^{kt}$, where y is the population after t minutes. When $t=0$, $y=200$, so $y = 200e^{kt}$. When $t=30$, $y=600$, so $3 = e^{k(30)}$, $30k = \ln 3$, or $k = \frac{\ln 3}{30}$. When $t=45$, $y = 200e^{\ln 3/30\cdot 45} = 200e^{3/2\ln 3} = 200(3)^{3/2} = \mathbf{600\sqrt{3}}$ **bacteria** (ignoring appropriate rounding).

8. **A** Let $y = \tan^{-1}4$. Then $\tan y = \frac{4}{1}$, so y is an acute angle of a right triangle with opposite side 4, adjacent side 1, hence hypotenuse $\sqrt{17}$. Therefore, $\cos y = \mathbf{\frac{1}{\sqrt{17}}}$.

9. **C** $f'(x) = 3x^2-3 = 0$ when $x=1$ or $x=-1$. Since f is continuous on a closed, bounded interval, it attains its absolute maximum and minimum at either a critical value or an endpoint. Since $f(-1)=3$, $f(1)=-1$, $f(3)=19$, the **minimum value is -1 and the maximum value is 19**.

10. **E** An antiderivative of $f(x) = \ln x$ means its derivative is $\ln x$. Using the product rule, $\frac{d}{dx}(x\ln x - x) = x\cdot\frac{1}{x} + 1\cdot\ln x - 1 = \ln x$, so the correct antiderivative is $\mathbf{x\ln x - x}$.

11. **B** The limit given is the definition of the derivative, so the correct answer is $\frac{d}{dx}(5^x) = (\ln \mathbf{5})\mathbf{5^x}$.

12. **E** $a(t) = v'(t) = 3t+2$, so $v(t) = \frac{3}{2}t^2+2t+C$. Since $v(0)=0$, $C=0$ and $v(t) = \frac{3}{2}t^2+2t$. Thus, $v(2) = \frac{3}{2}(2^2)+2(2) = \mathbf{10}$ **ft/sec**.

13. .

    (a) $f'(x) = x\cdot\frac{1}{1+x^2} + 1\cdot\arctan x - \frac{1}{2}\cdot\frac{2x}{1+x^2} = \frac{x}{1+x^2} + \arctan x - \frac{x}{1+x^2} = \arctan \mathbf{x}$.

    (b) $g'(x) = \frac{3+4e^{4x}}{3x+2+e^{4x}}$, so $g'(0) = \frac{3+4}{2+1} = \mathbf{\frac{7}{3}}$

14. .

    (a) $f'(x)=0$ when $x=2$. $f'(x)<0$ when $x<2$ and $f'(x)>0$ when $x>2$, so f is **increasing on $(\mathbf{2},\infty)$ and decreasing on $(-\infty, \mathbf{2})$**.

    (b) Based on the above answer, f has a **local minimum at $\mathbf{x=2}$**.

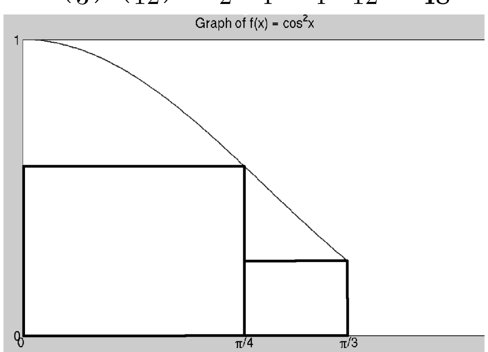
    (c) $f''(x) = (x-2)\cdot 3e^{3x} + 1\cdot e^{3x} = e^{3x}(3x-5)$. $f''(x)=0$ when $x=\frac{5}{3}$. $f''(x)>0$ when $x>\frac{5}{3}$ and $f''(x)<0$ when $x<\frac{5}{3}$. Therefore, f is **concave upward when $\mathbf{x>\frac{5}{3}}$ and concave downward when $\mathbf{x<\frac{5}{3}}$**.

15. Our goal is to minimize $C = 4lw + 2lh + 2wh$ under the restrictions that $l = w$ and $lwh = 8$, where l, w, and h are the dimensions of the box. Substituting $l = w$ into the other equations yields $C = 4w^2 + 4wh$ and $w^2h = 8$, or $h = \frac{8}{w^2}$. Thus, our goal is to minimize $C(w) = 4w^2 + 4w\left(\frac{8}{w^2}\right) = 4w^2 + \frac{32}{w}$ $(w > 0)$. $C'(w) = 8w - \frac{32}{w^2} = 0$ when $8w^3 - 32 = 0$, or $w = \sqrt[3]{4}$. By testing the signs of $C'(w)$ on either side or by showing $C''(\sqrt[3]{4}) > 0$, we see that this critical value is a local, and hence absolute minimum. Thus, $l = \sqrt[3]{4}$ and $h = \frac{8}{\sqrt[3]{4^2}} = 2\sqrt[3]{4}$. Therefore, the dimensions required are $\mathbf{\sqrt[3]{4} \times \sqrt[3]{4} \times 2\sqrt[3]{4}}$ **ft.**

16. .

(a) Area $\approx \sum_{i=1}^{n} f(x_i^*)\Delta x_i$. For the given partition, $n = 2$, so Area $\approx \boldsymbol{\sum_{i=1}^{2} \cos^2(x_i^*)\Delta x_i}$.

(b) Expanding and using the given partition yields Area $\approx \cos^2\left(\frac{\pi}{4}\right)\left(\frac{\pi}{4}\right) + \cos^2\left(\frac{\pi}{3}\right)\left(\frac{\pi}{12}\right) = \frac{1}{2}\cdot\frac{\pi}{4} + \frac{1}{4}\cdot\frac{\pi}{12} = \boldsymbol{\frac{7\pi}{48}}$

(c) 

17. Since the base approaches 1 and the exponent approaches ∞, rewrite using exponential and logarithms: $\lim_{x\to 0}(\cos x)^{1/x^2} = e^{\lim_{x\to 0}\ln((\cos x)^{1/x^2})}$ $= e^{\lim_{x\to 0}\frac{\ln(\cos x)}{x^2}}$ $= e^{\lim_{x\to 0}\frac{\frac{-\sin x}{\cos x}}{2x}}$ $= e^{\lim_{x\to 0}\frac{-\tan x}{2x}}$. Using L'Hospital's Rule again or the fact that $\frac{\tan x}{x} \to 1$ when $x \to 0$, the limit $= \mathbf{e^{-1/2}}$.