

Spring 2011 Math 151

Exam III Version B Solutions

1. **E** Since the numerator and denominator both approach 0, use L'Hospital's Rule:

$$\lim_{x \rightarrow 0} \frac{e^x - \cos x - 2x}{x^2 - 2x} = \lim_{x \rightarrow 0} \frac{e^x + \sin x - 2}{2x - 2} = \frac{1}{2}.$$

2. **A** Use properties of logarithms: $\ln(x^2 + x) = \ln(x+4)$, so $x^2 + x = x + 4$, $x^2 - 4 = 0$, which yields $x = 2$ or $x = -2$. Since the domain of the left-hand side of the original equation is $x > 0$, the only solution is $\mathbf{x = 2}$.

3. **A** f' positive means f is increasing, and f' decreasing means f'' is negative, so f is concave down. The only graph increasing and concave down is graph **A**.

4. **C** The original function f is decreasing when f' is negative, which occurs when $x \in (\mathbf{a}, \mathbf{c}) \cup (\mathbf{e}, \infty)$.

5. **E** f has a critical value when $f' = 0$, namely when $x = a, c, e$. Using the signs of the derivative f' , we find that f is increasing for $x \in (-\infty, a) \cup (c, e)$ and decreasing for $x \in (a, c) \cup (e, \infty)$. Therefore, f has a local minimum only when $\mathbf{x = c}$.

6. **D** Let $y = \log_4 \left(\frac{1}{8} \right)$. Then $4^y = \frac{1}{8}$, or $2^{2y} = 2^{-3}$, which means $2y = -3$ and $y = -\frac{3}{2}$. (Alternatively, $\log_4 \left(\frac{1}{8} \right) = \frac{\ln 1/8}{\ln 4} = \frac{-3 \ln 2}{2 \ln 2}$ and cancel)

7. **B** Exponential growth means $y = Ce^{kt}$, where y is the population after t minutes. When $t = 0$, $y = 200$, so $y = 200e^{kt}$. When $t = 30$, $y = 600$, so $3 = e^{k(30)}$, $30k = \ln 3$, or $k = \frac{\ln 3}{30}$. When $t = 45$, $y = 200e^{\ln 3/30 \cdot 45} = 200e^{3/2 \ln 3} = 200(3)^{3/2} = \mathbf{600\sqrt{3}}$ bacteria (ignoring appropriate rounding).

8. **B** Let $y = \tan^{-1} 4$. Then $\tan y = \frac{4}{1}$, so y is an acute angle of a right triangle with opposite side 4, adjacent side 1, hence hypotenuse $\sqrt{17}$. Therefore, $\cos y = \frac{1}{\sqrt{17}}$.

9. **E** $f'(x) = 3x^2 - 3 = 0$ when $x = 1$ or $x = -1$. Since f is continuous on a closed, bounded interval, it attains its absolute maximum and minimum at either a critical value or an endpoint. Since $f(-1) = 3$, $f(1) = -1$, $f(3) = 19$, the **minimum value is -1 and the maximum value is 19**.

10. **E** An antiderivative of $f(x) = \ln x$ means its derivative is $\ln x$. Using the product rule, $\frac{d}{dx}(x \ln x - x) = x \cdot \frac{1}{x} + 1 \cdot \ln x - 1 = \ln x$, so the correct antiderivative is $\mathbf{x \ln x - x}$.

11. **B** The limit given is the definition of the derivative, so the correct answer is $\frac{d}{dx}(5^x) = (\ln 5)\mathbf{5^x}$.

12. **B** $a(t) = v'(t) = 3t + 2$, so $v(t) = \frac{3}{2}t^2 + 2t + C$. Since $v(0) = 0$, $C = 0$ and $v(t) = \frac{3}{2}t^2 + 2t$. Thus, $v(2) = \frac{3}{2}(2^2) + 2(2) = \mathbf{10 \text{ ft/sec}}$.

13. .

$$(a) f'(x) = x \cdot \frac{1}{1+x^2} + 1 \cdot \arctan x - \frac{1}{2} \cdot \frac{2x}{1+x^2} = \frac{x}{1+x^2} + \arctan x - \frac{x}{1+x^2} = \arctan x.$$

$$(b) g'(x) = \frac{2 + 3e^{3x}}{2x + 4 + e^{3x}}, \text{ so } g'(0) = \frac{2 + 3}{4 + 1} = \mathbf{1}$$

14. .

- (a) $f'(x) = 0$ when $x = -2$. $f'(x) < 0$ when $x < -2$ and $f'(x) > 0$ when $x > -2$, so f is **increasing on $(-2, \infty)$ and decreasing on $(-\infty, -2)$** .

- (b) Based on the above answer, f has a **local minimum at $\mathbf{x = -2}$** .

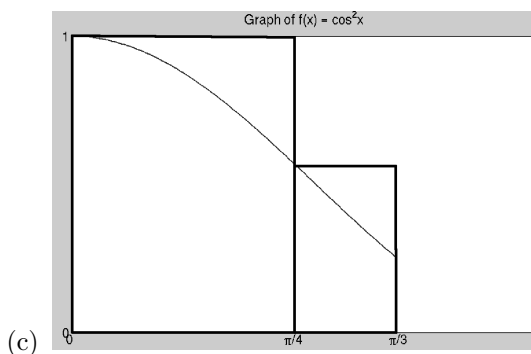
- (c) $f''(x) = (x + 2) \cdot 3e^{3x} + 1 \cdot e^{3x} = e^{3x}(3x + 7)$. $f''(x) = 0$ when $x = -\frac{7}{3}$. $f''(x) > 0$ when $x > -\frac{7}{3}$ and $f''(x) < 0$ when $x < -\frac{7}{3}$. Therefore, f is **concave upward when $\mathbf{x > -\frac{7}{3}}$ and concave downward when $\mathbf{x < -\frac{7}{3}}$** .

15. Our goal is to minimize $C = 4lw + 2lh + 2wh$ under the restrictions that $l = w$ and $lwh = 8$, where l , w , and h are the dimensions of the box. Substituting $l = w$ into the other equations yields $C = 4w^2 + 4wh$ and $w^2h = 8$, or $h = \frac{8}{w^2}$. Thus, our goal is to minimize $C(w) = 4w^2 + 4w\left(\frac{8}{w^2}\right) = 4w^2 + \frac{32}{w}$ ($w > 0$). $C'(w) = 8w - \frac{32}{w^2} = 0$ when $8w^3 - 32 = 0$, or $w = \sqrt[3]{4}$. By testing the signs of $C'(w)$ on either side or by showing $C''(\sqrt[3]{4}) > 0$, we see that this critical value is a local, and hence absolute minimum. Thus, $l = \sqrt[3]{4}$ and $h = \frac{8}{\sqrt[3]{4^2}} = 2\sqrt[3]{4}$. Therefore, the dimensions required are $\sqrt[3]{4} \times \sqrt[3]{4} \times 2\sqrt[3]{4}$ ft.

again or the fact that $\frac{\tan x}{x} \rightarrow 1$ when $x \rightarrow 0$, the limit = $e^{-1/2}$.

16. .

- (a) Area $\approx \sum_{i=1}^n f(x_i^*)\Delta x_i$. For the given partition, $n = 2$, so Area $\approx \sum_{i=1}^2 \cos^2(x_i^*)\Delta x_i$.
- (b) Expanding and using the given partition yields Area $\approx \cos^2(0)\left(\frac{\pi}{4}\right) + \cos^2\left(\frac{\pi}{4}\right)\left(\frac{\pi}{12}\right) = 1 \cdot \frac{\pi}{4} + \frac{1}{2} \cdot \frac{\pi}{12} = \frac{7\pi}{24}$



17. Since the base approaches 1 and the exponent approaches ∞ , rewrite using exponential and logarithms:

$$\begin{aligned} \lim_{x \rightarrow 0} (\cos x)^{1/x^2} &= e^{\lim_{x \rightarrow 0} \ln((\cos x)^{1/x^2})} \\ &= e^{\lim_{x \rightarrow 0} \frac{\ln(\cos x)}{x^2}} = e^{\lim_{x \rightarrow 0} \frac{-\frac{\sin x}{\cos x}}{2x}} \\ &= e^{\lim_{x \rightarrow 0} \frac{-\tan x}{2x}}. \quad \text{Using L'Hospital's Rule} \end{aligned}$$