Exam II Solutions (Form B)

1. A \( \frac{d}{dx}(2 \cos x - \sin x) = -2 \sin x - \cos x \)

2. B \( y' = 1 - \sec x \tan x \); when \( x = 0 \), \( y' = 1 - \sec 0 \tan 0 = 1 \)

3. A \( f'(x) = 8(3x - 1)^7(3) \) using the Chain Rule, so \( f'(0) = 8(-1)^7(3) = -24 \)

4. E The derivatives of \( \sin x \) follow the pattern \( \cos x, -\sin x, -\cos x, \sin x, \ldots \) so the eleventh derivative is the same as the third derivative, \( -\cos x \).

5. D \( r'(t) = < 1/3 t^{-2/3}, 2e^{2t} > \) so \( r'(1) = < 1/3, 2e^2 > \)

6. C Newton’s Method says that
   \[ x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 2 - \frac{2^3 - 2 \cdot 2 - 5}{3(2^2) - 2} = 2 - \frac{1}{10} = 21/10 \]

7. A Begin with the exponent: \( \lim_{x \to 0} \frac{1}{x} = -\infty \). Thus \( \lim_{x \to 0} 3^{1/x} = 0 \) and \( \lim_{x \to 0} \frac{1}{2 + 3^{1/x}} = \frac{1}{2} + 0 = \frac{1}{2} \)

8. A The graphs of the functions are shown below. Using the horizontal line test, \( f_1 \) and \( f_2 \) are NOT one-to-one.

9. A Either use the formula \( L(x) = f(a) + f'(a)(x - a) \) or just find the equation of the tangent line. \( f(\pi) = e^{\sin \pi} = e^0 = 1 \). \( f'(x) = e^{\sin x} \cos x \), so \( f'(\pi) = e^{\sin \pi} \cos \pi = -1 \). The equation is \( L(x) = 1 + (-1)(x - \pi) = \pi + 1 - x \).

10. C Switch \( x \) and \( y \) and solve for \( y \): \( x = \frac{2}{1+y^2}, x+xy^2 = 2, xy^2 = 2-x, y = \sqrt{\frac{2-x}{x}} \) (we take the positive square root since the domain of the original function is \([0, \infty)\)). The domain of this function is when the fraction is positive, which occurs when \( x \in (0, 2) \).

11. E The limit is nothing more than the definition of \( f'(a) \), where \( f(x) = \tan^2 x \) and \( a = \frac{\pi}{4} \).

12. Using the Product Rule, \( y' = 2xe^x + x^2e^x = (2x + x^2)e^x \), \( y'' = (2 + 2x)e^x + (2x + x^2)e^x = (2 + 4x + x^2)e^x \), \( y''' = (4 + 2x)e^x + (2 + 4x + x^2)e^x = (6 + 6x + x^2)e^x \).

13. Using all derivatives into the differential equation yields \( (6 + 6x + x^2)e^x - 3(2 + 4x + x^2)e^x + 3(2 + x^2)e^x - x^2e^x = (6 + 6x + x^2 - 6 - 12x - 3x^2 + 6x + 3x^2 - x^2)e^x = 0 \)

14. Moving the terms with \( y' \) to the left and terms without \( y' \) to the right yields \( y'(x-1)^{1/2} + (1/2)xy'(y-1)^{-1/2} - 3xy' = 3y - (1/2)yx(x-1)^{-1/2} - (y-1)^{1/2} \), so \( y' = \frac{3y - (1/2)yx(x-1)^{-1/2} - (y-1)^{1/2}}{-3x + (1/2)yx(y-1)^{-1/2} + (x-1)^{1/2}} \).
14. i) The relating formula is \( V = x^3 \), given \( \frac{dV}{dt} = -2 \). At the instant when \( V = 64 \), we know \( x = 4 \).

Differentiate and substitute: \( \frac{dV}{dt} = 3x^2 \frac{dx}{dt} \), \(-2 = 3(4)^2 \frac{dx}{dt} \), or \( \frac{dx}{dt} = -\frac{1}{24} \) \( m \) \( \text{min}^{-1} \)

ii) The relating formula is \( S = 6x^2 \). Differentiate: \( \frac{dS}{dt} = 12x \frac{dx}{dt} \). At the instant when \( x = 4 \), \( \frac{dx}{dt} = -\frac{1}{24} \), so substitute: \( \frac{dS}{dt} = 12x \frac{dx}{dt} \), \( \frac{dx}{dt} = -\frac{1}{24} \) \( m \) \( \text{min}^{-1} \).

15. i) Using the quotient rule \( x'(t) = \frac{(1 + t)(-1) - (1 - t)(1)}{(1 + t)^2} = \frac{-2}{(1 + t)^2} \)

ii) Using the quotient rule \( g'(t) = \frac{(1 + t)(-1/2)t^{-1/2} - (1 - t^{1/2})(1)}{(1 + t)^2} \)

iii) The slope is found by \( \frac{dy}{dx} = \frac{dy/dt}{dx/dt} \). When \( x = 0 \) and \( y = 0 \), we have \( t = 1 \), so the slope is \( \frac{dy}{dx} \bigg|_{t=1} = \frac{(1+1)(-1/2)-1}{(1+1)^2} = \frac{1}{2} \)

16. The slope of the tangent line is given by \( g'(1) = \frac{1}{f'(g(1))} \). \( f'(x) = e^{x^3+3x}(3x^2 + 3) \). To find \( g(1) \), solve \( f(x) = 1 \). \( e^{x^3+3x} = 1 \) when \( x^3 + 3x = x(x^2 + 3) = 0 \), or \( x = 0 \) (Since \( x^2 + 3x \neq 0 \)).

So \( g'(1) = \frac{1}{f'(0)} = e^{0}(3 \cdot 0 + 3) = \frac{1}{3} \). Since \( g(1) = 0 \), the tangent line passes through the point \( (1, 0) \). Therefore, the equation is \( y - 0 = \frac{1}{3}(x - 1) \), or \( y = \frac{1}{3}x - \frac{1}{3} \).