Math 151 Fall 2009 Exam I
Solutions-Form A

1. A: The vector \(\langle 2, -3 \rangle\) is parallel to the line \(x = 2t + 1, y = -3t + 5\). Hence \(\langle 3, 2 \rangle\) is perpendicular to the line. Divide by the magnitude to make the vector a unit vector. \(\frac{\langle 3, 2 \rangle}{\sqrt{13}} = \left\langle \frac{3}{\sqrt{13}}, \frac{2}{\sqrt{13}} \right\rangle\).

2. D: \(\lim_{x \to -1} f(7) = \lim_{x \to -1} 7 = 7\). B: To test continuity, we first note that \(2x + 1\) is continuous for \(x < -1\), \(x^2 - 3\) is continuous for \(-1 < x \leq 2\) and \(\frac{1}{x} + \frac{1}{2}\) is continuous for \(x > 2\). Thus we need only to check continuity for \(x = -1\) and \(x = 2\).

3. E: Use the product rule to differentiate \(W = (8 \text{ pounds})(25 \text{ feet})(\cos 60^\circ)\). Since \(f'(0) = 2\) and \(f'(1) = 2\), there is a solution to \(f(c) = 0\) on the interval \((0, 1)\).

4. D: Use the quotient rule to differentiate \(f(x) = x^2 + 1\). Hence the slope of the tangent line is \(m = f'(2) = \frac{1}{9}\). Also, \(\theta = 60^\circ\). Hence \(W = 100\) foot pounds.

5. B: Use the quotient rule to differentiate \(f(x) = \frac{x}{1 + x}\).

6. C: \(x = \sin t\) and \(y = 4 + \cos t\) is equivalent to \(x = \sin t, y - 4 = \cos t\). Since \(\sin^2 t + \cos^2 t = 1\), \(x^2 + (y - 4)^2 = 1\).

7. B: To test \(f(x) = \begin{cases} 2x + 1 & \text{if } x \leq -1 \\ x^2 - 3 & \text{if } -1 < x \leq 2 \\ \frac{1}{x} + \frac{1}{2} & \text{if } x > 2 \end{cases}\) for continuity, we first note that \(2x + 1\) is continuous for \(x < -1\), \(x^2 - 3\) is continuous for \(-1 < x < 2\) and \(\frac{1}{x} + \frac{1}{2}\) is continuous for \(x > 2\). Thus we need only to check continuity for \(x = -1\) and \(x = 2\).

8. B: \(\lim_{x \to -3} \frac{x - 1}{x^2(x + 3)} = -\infty\) since \(x = -3\) is a vertical asymptote and \(\frac{x - 1}{x^2(x + 3)} < 0\) for \(x > -3\).

9. D: First, note \(f(x) = x^3 - x^2 + 3x - 1\) is continuous, so we will apply the Intermediate Value Theorem. Since \(f(0) = -1 < 0\) and \(f(1) = 2 > 0\), there is a solution to \(f(c) = 0\) on the interval \((0, 1)\).

10. C: \(f(x) = |x^2 - 16|\) is not differentiable at \(x = \pm 4\) since \(f'(4)\) and \(f'(-4)\) do not exist. (See figure below)

11. Multiply by the conjugate:

\[
\lim_{x \to -\infty} \frac{x + \sqrt{x^2 + 3x}}{x - \sqrt{x^2 + 3x}} - \frac{x - \sqrt{x^2 + 3x}}{x - \sqrt{x^2 + 3x}}
\]

\[
= \lim_{x \to -\infty} \frac{-3x}{x - \sqrt{x^2 + 3x}} = \lim_{x \to -\infty} \frac{-3x}{x - (\sqrt{x^2 + 3x})/x} = \lim_{x \to -\infty} \frac{-3}{1 + \sqrt{1 + 3/x}} = \frac{3}{2}
\]
12. (i) To find the cosine of the angle between the vectors \( \langle 1, 4 \rangle \) and \( \langle 2, 3 \rangle \), we will use the formula

\[
\cos \theta = \frac{\langle 1, 4 \rangle \cdot \langle 2, 3 \rangle}{|\langle 1, 4 \rangle||\langle 2, 3 \rangle|}
\]

\[
\cos \theta = \frac{2 + 12}{\sqrt{17} \sqrt{13}} = \frac{14}{\sqrt{221}}
\]

(ii) Let \( a = \langle 2, 3 \rangle \) and \( b = \langle 1, 4 \rangle \). Then

\[
\text{comp}_a b = \frac{\langle 2, 3 \rangle \cdot \langle 1, 4 \rangle}{|\langle 2, 3 \rangle|} = \frac{14}{\sqrt{13}}
\]

(iii) \( \text{proj}_a b = \frac{\langle 2, 3 \rangle \cdot \langle 1, 4 \rangle}{|\langle 2, 3 \rangle|^2} \langle 2, 3 \rangle \)

\[
= \frac{14}{13} \langle 2, 3 \rangle
\]

\[
= \left< \frac{28}{13}, \frac{42}{13} \right>
\]

13. \( f(x) = \frac{2}{x - 3} \). 

\[ f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h} \]

\[ = \lim_{h \to 0} \frac{2}{x + h - 3} - \frac{2}{x - 3} \]

\[ = \lim_{h \to 0} \frac{2(x - 3) - 2(x + h - 3)}{h(x + h - 3)(x - 3)} \]

\[ = \lim_{h \to 0} \frac{-2h}{h(x + h - 3)(x - 3)} \]

\[ = \frac{-2}{(x - 3)^2} \]

14. Since \( |F_1| = 8 \) pounds and \( |F_2| = 10 \) pounds,

\[ F_1 = \langle -8 \cos 45^\circ, 8 \sin 45^\circ \rangle = \langle -4\sqrt{2}, 4\sqrt{2} \rangle. \]

\[ F_2 = \langle 10 \cos 30^\circ, 10 \sin 30^\circ \rangle = \langle 5\sqrt{3}, 5 \rangle. \]

Thus the resultant force is

\[ F = F_1 + F_2 = \langle -4\sqrt{2} + 5\sqrt{3}, 4\sqrt{2} + 5 \rangle. \] Thus the magnitude of the resultant force is

\[ |F| = \sqrt{(-4\sqrt{2} + 5\sqrt{3})^2 + (4\sqrt{2} + 5)^2} \text{ pounds} \]

15. a.) Since \( x < 2, |x - 2| = -(x - 2) \). Thus

\[ \lim_{x \to 2^-} \frac{x - 2}{x - 2} = \lim_{x \to 2^-} \frac{x - 2}{-(x - 2)} = -1 \]

b.) To find the value of \( a \) for which \( \lim_{x \to 1} \frac{3x^2 + ax + a + 3}{x^2 + x - 2} \) exists, we use the fact that, in order for the limit to exist at \( x = 1 \), \( 3x^2 + ax + a + 3 \) must have a factor of \( x - 1 \) in order to eliminate the division by zero. Hence if \( x - 1 \) is a factor of \( 3x^2 + ax + a + 3 \), it follows that

\[ 3(1)^2 + a(1) + a + 3 = 0 \]

Thus \( 3 + a + a + 3 = 0 \) yielding \( a = -3 \).