Spring 2010 Math 151

Exam III Version A Solutions

1. **B** \( y = \log_9 27 \) is equivalent to \( 9^y = 27 \), or \( 3^{2y} = 3^3 \), so \( 2y = 3 \) and \( y = \frac{3}{2} \).

2. **C** Using properties of logarithms, we have \( \log_{10}((5 - x)(2 - x)) = 1 \), which is equivalent to \( 10 - 7x + x^2 = 10^1 \), or \( x^2 - 7x = 0 \). Then \( x(x - 7) = 0 \), or \( x = 0 \) and \( x = 7 \). A quick check shows that \( x = 7 \) is not in the domain of the original equation, so the only solution is \( x = 0 \).

3. **D** \( y = \sin^{-1}\left(\frac{2}{x}\right) \) means \( \sin y = \frac{2}{x} \). From the reference triangle below, we see that \( \cos y = \frac{\sqrt{x^2 - 4}}{x} \).

4. **B** \( f'(x) = \frac{3x^2}{1+(x^3)^2} = \frac{3x^2}{1+x^6} \), so \( f'(1) = \frac{3}{2} \).

5. **E** The limit is of the indeterminate form \( 0 \cdot 0 \), so apply L’Hospital’s Rule:
   \[
   \lim_{x \to 0} \frac{x^4 - 1}{x^2} = \lim_{x \to 0} \frac{4x^3}{2x} = \frac{2}{3}.
   \]

6. **E** The limit is of the indeterminate form \( \infty \cdot 0 \), so rewrite as a fraction:
   \[
   \lim_{x \to (\pi/2)^-} \sec(3x)\cos(x) = \lim_{x \to (\pi/2)^-} \frac{\cos(x)}{\cos(3x)} = \lim_{x \to (\pi/2)^-} \frac{1}{-3\sin(3x)} = \frac{-1}{3}.
   \]

7. **B** \( f \) is decreasing when \( f' \) is negative, which is on the interval \((-\infty, -5) \cup (4, \infty)\).

8. **A** \( f \) is concave down when \( f'' \) is negative, or when \( f' \) is decreasing, which is on the interval \((0, \infty)\).

9. **D** \( f'(x) = 3(x - 3)^2 = 0 \) when \( x = 3 \). Testing the endpoints and the critical value in the original function yield \( f(1) = -8 \), \( f(3) = 0 \), \( f(4) = 1 \), so the absolute minimum is \(-8 \) when \( x = 1 \).

10. **C** Apply the power rule to the first term and the logarithm rule to the second (NOTE the power rule for \( x^n \) only applies when \( n \neq -1 \)).
   \[
   4 \cdot \frac{3}{5} x^{5/3} + 3 \ln |x| = \frac{12}{5} x^{5/3} + 3 \ln |x|.
   \]

11. **C** Expand the sum:
   \[
   \sum_{i=1}^{4} i^2 = 1^2 + 2^2 + 3^2 + 4^2 = 1 + 4 + 9 + 16 = 30.
   \]

12. **E** Since there are only 2 subintervals, the area = \( \sum_{i=1}^{2} f(x_i^*) \Delta x_i = f(0)(2) + f(4)(4) = (20)(2) + (4)(4) = 56 \).

13. \( y = x^{\sin x} \cos x \), so \( \ln y = \ln(x^{\sin x} \cos x) = \sin x \ln x + \ln(\cos x) \). Differentiate implicitly:
   \[
   \frac{y'}{y} = \cos x \ln x + \sin x \frac{\sin x}{x} - \frac{\sin x}{\cos x} \Rightarrow y' = (x^{\sin x} \cos x) \left( \cos x \ln x + \frac{\sin x}{x} - \tan x \right).
   \]

14. Let \( y \) = the amount of Polonium-218 (in mg) after \( t \) days. Then \( y = y_0 e^{kt} = 1000e^{kt} \). Substitute the amount after 5 days and solve for \( k \):
   \[
   300 = 1000e^{k(5)} \Rightarrow \frac{3}{10} = e^{5k} \Rightarrow 5k = \ln \left( \frac{3}{10} \right) \Rightarrow k = \frac{1}{5} \ln \left( \frac{3}{10} \right).
   \]
   After 10 days \( 1000e^{1/5 \ln(3/10) + 10} = 1000e^{2 \ln(3/10)} = 1000 \left( \frac{3}{10} \right)^2 = 90 \) mg remain.

15. **a** Find the critical values from the given \( f' \):
   \( (x^2 - 3)e^x = 0 \) when \( x = \pm \sqrt{3} \). Testing each subinterval, we find \( f' \) is positive on \(( -\infty, -\sqrt{3} ) \cup ( \sqrt{3}, \infty ) \) and \( f' \) is negative on \(( -\sqrt{3}, \sqrt{3} ) \). Therefore, \( f \) is increasing on \(( -\infty, -\sqrt{3} ) \cup ( \sqrt{3}, \infty ) \) and decreasing on \(( -\sqrt{3}, \sqrt{3} ) \).

   **b** Based on the intervals of increase and decrease, \( f \) has a relative maximum at \( x = -\sqrt{3} \) and a relative minimum at \( x = \sqrt{3} \).
(c) \( f''(x) = 2xe^x + (x^2 - 3)e^x = (x^2 + 2x - 3)e^x = (x + 3)(x - 1)e^x \), which is 0 when \( x = -3 \) or \( x = 1 \). Testing each subinterval, we find \( f'' \) is positive on \((-\infty, -3) \cup (1, \infty)\) and \( f'' \) is negative on \((-3, 1)\). Therefore, \( f \) is concave up on \((-\infty, -3) \cup (1, \infty)\) and concave down on \((-3, 1)\).

16. Let \( \ell \) be the length of the field (horizontal) and \( w \) be the width. Our goal is to maximize \( A = \ell w \) with the restriction that \( 2\ell + 3w = 3000 \). Solve the second equation for \( \ell \) and substitute into the first: \( \ell = -\frac{3}{2}w + 1500 \), \( A = -3w + 1500 = 0 \) when \( w = 500 \). \( A'' = -3 \), which shows \( A \) has a maximum at \( w = 500 \), \( \ell = -\frac{3}{2}(500) + 1500 = 750 \). The dimensions which maximize area are \( 750 \times 500 \) ft.

17. Find the antiderivative of each component to obtain \( v(t) = \langle e^t + \sin t, e^t - \cos t \rangle + C \). If \( t = 0 \), then \( v(0) = \langle 1, 1 \rangle = \langle e^0 + \sin 0, e^0 - \cos 0 \rangle + C = \langle 1, 0 \rangle + C \), so \( C = \langle 0, 1 \rangle \). Thus \( v(t) = \langle e^t + \sin t, e^t - \cos t + 1 \rangle \). Find the antiderivative again to obtain \( r(t) = \langle e^t - \cos t, e^t - \sin t + t + 1 \rangle \). If \( t = 0 \), then \( r(0) = \langle 0, 0 \rangle = \langle e^0 - \cos 0, e^0 - \sin 0 + 0 \rangle + C = \langle 0, 1 \rangle + C \), so \( C = \langle 0, -1 \rangle \). Therefore, \( r(t) = \langle e^t - \cos t, e^t - \sin t + t - 1 \rangle \).